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DYNAMIC MODELING OF A SINGLE-LINK FLEXIBLE MANIPULATOR ROBOT WITH TRANSLATIONAL AND ROTATIONAL MOTIONS

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ABSTRACT: The flexible manipulator is widely used in space robots, robot arm, and manufacturing industries that produce micro-scale products. This study aims to formulate the equation of motion of a flexible single-link manipulator system that moves translationally and rotationally and to develop computational codes with finite element methods in performing dynamic simulation on the vibration of the flexible manipulator system. The system of the single-link flexible manipulator (SLFM) consists of the aluminum beam as a flexible link, clamp part to hold the link, DC motor to rotate drive shaft, a trajectory to transfer link in translational motion, and servo motor to rotate link. Computational codes in time history response (THR) and Fast Fourier Transform (FFT) processing were developed to identify the dynamic behavior of the link. The finite element-method and Newmark-beta are used in simulating the SLFM. Simulation using the finite element method has displayed dynamic behavior through a graph of FFT on free vibration and THR graph on forced vibration by the excitation force due to the translational and rotational motions of the system. In the simulation of free vibration, the natural frequency of the system is 8.3 [Hz].

KEY WORDS: *Dynamic modeling, single-link flexible manipulator, finite-element method, time history response, translational and rotational motions.*

1. INTRODUCTION

In the industrial application and robotic systems, the single-link flexible manipulator (SLFM) are expected to work optimally during operation. Some researchers who study flexible manipulator for space are found in many reputable journals such as [1]-[6]. For the application of space robot, manipulator robot is designed to lift relatively small objects such as rock samples and remove existing obstacles so that the manipulator must be able to carry out operations with better positioning accuracy. Lighter weight aims to save load capacity and reduce the cost of rocket launches and less energy consumption. In fact, SLFM has limitations with frequent vibrations that cause system performance to be interrupted. Therefore, further research is needed to describe the dynamic conditions of the SLFM. Several studies have investigated the performance and control system of the SLFM. Qiu [7] in his study reviewed two main parts of the flexible Cartesian manipulator (FCM), namely the control algorithm and validating it with the experiment. Fiberglass colophony material was used of the FCM. FCM experiment only displayed translational motion along the *Y*-axis (*O-Y-O*). In vice versa, Shin and Rhim [8] conducted modeling with the Newtonian approach to horizontal translational motion (*O-X-O*) on the flexible link. The modeling result showed the effect of lateral vibration and dynamic stiffness at varying frequencies of the beam. Yang et al [9] have designed an observation using partial differential equations (PDE) using kinetic energy and potential energy methods to predict vibration, unlimited dimensional condition by only requiring practical measurement values from boundary position. From the simulation result, the vibration can be removed using PD control.

The research is somewhat different done by Ata et al [10] who approached Euler Bernoulli on SLFM. In this approach, a flexible beam is considered rigid marked by a straight line that extends from point O to the end of the beam, which they call virtual link (VL) which is perpendicular to the tangent vector line. Beam on VL moves rotationally. Still on the SLFM, Muhammad et al [12]-[15] carried out comprehensive simulations and experiments with the FEM, PD, and AF control approaches to vibration on the SLFM. They [16]-[18] also added a piezoelectric actuator on aluminum-based links, which aim to generate stress on two Degrees of Freedom (DOF). Meanwhile, Mahto [18], still in the rotation movement of the SLFM, has investigated it using the Finite element method (FEM) approach with five DoF assuming the link manipulator as Euler-Bernoulli beam.

A myriad of investigations on the flexible manipulator using FEM in analyzing vibration has been reported in the literature such as basic modeling replacement in control algorithm [19], multi-link flexible robotic manipulator with a smart piezoelectric transducer [20], vibration control of a single-link flexible composite manipulator [21]. Finite-element modeling with piezo-integrated structure using Hamiltonian principles [22], and free vibration analysis on thin-plate [23]. Based on several studies, in general, they researched of the SLFM system with just one motion, translation or rotational. Whereas if the SLFM is applied in industry, it generally uses two movements simultaneously, namely translational and rotational motions. The problem is the lack of research that combines translational and rotational motions of the SLFM system, which result in inaccurate positioning. Therefore, this research is very interesting to do a more in-depth study.

On the other hand, several similarities were also identified such as the use of the Lagrange's equation, kinetic energy, and potential energy are used to derive the equation of motion [24]. The difference is, Bien's work focuses on the combination of translational and rotational motion of a two-link manipulator, where the first link moves horizontally, and the second link moves rotationally. Research from Muhammad et al [17] as the main reference in this research that has formulated the equation of rotational motion at SLFM becomes interesting to be developed by combining translational and rotational motion.

The purpose of this study is to formulate the equations of motion and compose computational codes by FEM on the SLFM system that moves translationally and rotationally. This research used FEM in carrying out dynamic simulations of the SLFM system. The results of the simulation showed the vibration dynamics of the system by displaying a Time History Response (THR) and Fast Fourier Transform (FFT) processing.

2. Dynamic Modeling

Based on the kinematic model of the SLFM that moves translationally as shown in Fig. 1. The position vector of point P moves translationally at the O-Y coordinate. The link moves translationally on the horizontal direction at the coordinates O-Yp as far as Y. The base of the link experiences a positive displacement of the Y-axis as far as O-y, the positive X-axis at position $X_p = x_p$ and the end of the link is a position vector P. The position vector r (x,t) at point P on the link at $t = t$, is represented in the O-X-Y coordinates.

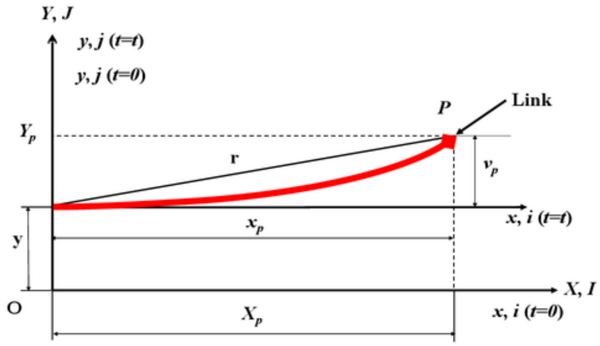


Fig. 1. The position vector of point P on the link that moves translationally

The position vector of point P in the O-X-Y coordinates is

$$r(x,t) = X_p(x,t)\boldsymbol{I} + Y_p(x,t)\boldsymbol{J} \quad (1)$$

Where;

$$X_p(x,t) = x_p \quad (2)$$

$$Y_p(x,t) = y + v_p \quad (3)$$

The vector velocity to point P in the coordinates O-x-y is given with

$$\dot{\boldsymbol{r}}(x,t) = \dot{X}_p(x,t)\boldsymbol{I} + \dot{Y}_p(x,t)\boldsymbol{J} \quad (4)$$

2.1. Equation of motion using Finite-element method

This paper uses the finite element method approach, where each partition on the link is divided into several elements. The aim is to find out the mass matrix (M) and the stiffness matrix (K) which will be used to simulate the dynamics of the system. The system dynamics model is a cantilever structure which is partitioned into six elements. Fig. 2 shown the Translational coordinate X-Y divided by one-dimension and two-node elements with four boundary conditions together at node i and i+1.

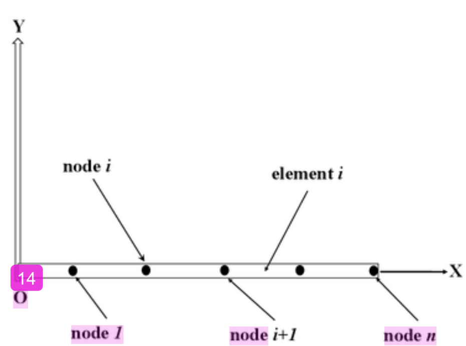


Fig. 2. Translational coordinate of the link

An element of the link is shown in fig. 3. Finite element has two DOF, namely the lateral deformation $v(t)$ and slope $\Psi(t)$. The physical properties of the system consist of the length of the link (Li), the cross-sectional area of the link (Si), and the moment of inertia (Ii). Each element has mechanical properties namely Young's modulus and mass density which is denoted by Ei and ρi.

The nodal displacement vector expressed by

$$\delta_i = [v_i, \varphi_i, v_{i+1}, \varphi_{i+1}] \tag{5}$$

With

$$v_i = a_1 + a_2 x_i + a_3 {x_i}^2 + a_4 {x_i}^3 \tag{6}$$

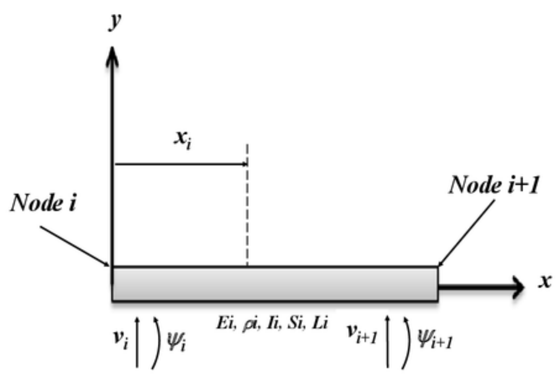


Fig. 3. Nodal displacement of the link

The discretization process, equation of translational motion can be obtained as follows;

Acceleration of P can be written by

$$\dot{\boldsymbol{r}}^T . \dot{\boldsymbol{r}} = {\dot{x}_p}^2 + \dot{y}^2 + {\dot{v}_p}^2 + 2\dot{y}\dot{v}_p \tag{7}$$

The general articulation of kinetic energy (T), with V being the velocity can be expressed by

$$\boldsymbol{T} = \frac{1}{2}\int_v \rho . V^2 dv \tag{8}$$

Kinetic energy (T_i) of the system is given by

$$\boldsymbol{T_i} = \frac{1}{2}\int_{v_i} \rho_i . \dot{\boldsymbol{r}}^T . \dot{\boldsymbol{r}} . dv_i \tag{9}$$

So that kinetic energy of translation motion ($T_i t$) for the system can be written as

$$T_i t = \frac{1}{2} m_i {\dot{x}_p}^2 + \frac{1}{2} m_i \dot{y}^2 + \frac{1}{2} {\dot{\delta}_i}^T M_i \dot{\delta}_i + \dot{y} {f_{t_i}}^T \dot{\delta}_i \tag{10}$$

where

$$\dot{y} {f_{t_i}}^T \dot{\delta}_i = \frac{m_i}{12} [6 \quad l_i \quad 6 \quad -l_i] \tag{11}$$

The potential energy (U_i) of the system is written in the equation

$$U_i = \frac{1}{2} {\delta_i}^T K_i \delta_i \tag{12}$$

The dissipation function (R_i) of the system is given with

$$R_i = \frac{1}{2} C_i \dot{\delta}_i^{\,2} \tag{13}$$

The mathematical model of dynamic system using the Lagrange's equation is written with

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial \dot{q}_k} + \frac{\partial U}{\partial \dot{q}_k} + \frac{\partial R}{\partial \dot{q}_k} = Q_k \tag{14}$$

If we substitute variables from Eq. (14) as follows

$$\frac{d}{dt}\left(M_i \dot{\delta}_i - \dot{y}(t) {f_{ti}}^T\right) + K_i \delta_i + \frac{\partial R}{\partial \dot{q}_k} + C_i \dot{\delta}_i = Q_k \tag{15}$$

Furthermore,

$$M_i \ddot{\delta}_i - \ddot{y}(t) {f_{ti}}^T + C_i \dot{\delta}_i + K_i \delta_i = 0 \tag{16}$$

So that the equation of motion for the *i-th* element is given with

$$M_i \ddot{\delta}_i + C_i \dot{\delta}_i + K_i \delta_i = \ddot{y}(t) {f_{ti}}^T \tag{17}$$

Where M_i, C_i, K_i, and $\ddot{y}(t){f_{ti}}^T$ are the mass matrix, damping matrix, stiffness matrices, and excitation forces of the DC motor respectively. Matrix M_i, C_i, K_i is an element of the equation contained in Eq. (17) is a representation of;

$$M_i = \frac{\rho_i S_i L_i}{420} \begin{bmatrix} 156 & 22L_i & 54 & -13L_i \\ 22L_i & 4{L_i}^2 & 13L_i & -3{L_i}^2 \\ 54 & 13L_i & 156 & -22L_i \\ -13L_i & -3{L_i}^2 & -22L_i & 4{L_i}^2 \end{bmatrix} \tag{18}$$

$$K_i = \frac{E_i I_i}{{L_i}^3} \begin{bmatrix} 12 & 6L_i & -12 & 6L_i \\ 6L_i & 4{L_i}^2 & -6L_i & 2{L_i}^2 \\ -12 & -6L_i & 12 & -6L \\ 6L_i & 2{L_i}^2 & -16L_i & 4{L_i}^2 \end{bmatrix} \tag{19}$$

$$C_i = \alpha . K_i \tag{20}$$

While the translational motion of the force vector can be written with;

$$f_{ti}^T = -\frac{\rho_i S_i L_i}{12}\{6,\ l_i,\ 6, -l_i\} \tag{21}$$

The length of the *i-th* element is the length from element 1 to *i*, with the symbols l_i and l_{1+i}. Equation of translational motion of the SLFM with *i-th* element based on boundary conditions is written with;

$$\boldsymbol{M_n \ddot{\delta}_n + C_n \dot{\delta}_n + K_n \delta_n = \ddot{y}(t) f_{t_n}} \tag{22}$$

The kinetic energy of rotational motion ($T_i r$) of the system has been formulated in [11] as follows

$$T_i r = \frac{7}{6} m_i {l_i}^2 \dot{\theta}^2 + \frac{1}{2} \dot{\delta}_i^{\,T} M_i \dot{\delta}_i + \frac{1}{2} {\delta_i}^T \dot{\theta}^2 M_i \delta_i + \dot{\theta} {f_{r_i}}^T \dot{\delta}_i \tag{23}$$

And then the rotational force vector is written with

$$\ddot{\theta}(t) f_n = \frac{\rho_i S_i L_i}{60}\{30 l_{1-i} + 9 l_i,\ 5 l_{1-i}\, l_i + 2{l_i}^2,\ 21 l_i, -5 l_{1-i}\, l_i + 3{l_i}^2\} \tag{24}$$

Equation of motion of the *i*-element is written by[12]

$$M_n \ddot{\delta}_n + C_n \dot{\delta}_n + (K_n - \dot{\theta}^2(t) M_n)\delta_n = \ddot{\theta}(t) f_n \tag{25}$$

Where $\ddot{\theta}(t){f_{ri}}^T$ the respectively excitation forces.

To find out the kinetic energy of the system, kinetic energy caused by translational and rotational motions is obtained by combining kinetic energy from translational motion and rotational motion so that the kinetic energy of translational and rotational motions ($T_i tr$) of the system will be obtained;

$$T_i tr = T_i t + T_i r \tag{26}$$

From the discretization result of the kinetic energy of translational and rotational motions, then by substituting Eq. (10) and (23) into Eq. (26) is given by;

$$T_i tr = \frac{5}{3} m_i\left(\dot{x}_p{}^2 + \dot{y}^2 + l_i{}^2\dot{\theta}^2\right) + \dot{\delta}_i{}^T M_i \dot{\delta}_i - \frac{1}{2}\delta_i{}^T \dot{\theta}^2 M_i \delta_i - \left(\dot{y}{f_{t_i}}^T + \dot{\theta}{f_{r_i}}^T\right)\dot{\delta}_i \tag{27}$$

Applying Lagrange's equation, then substitution Eq. (27) to Eq. (14) will be obtained

$$\frac{d}{dt}\left(2M_i\dot{\delta}_i - \left(\dot{y}{f_{t_i}}^T + \dot{\theta}{f_{r_i}}^T\right)\right) - \dot{\theta}^2 M_i \delta_i + K_i\delta_i + \quad C_i\dot{\delta}_i = 0 \tag{28}$$

$$2M_i\ddot{\delta}_i - \left(\ddot{y}{f_{t_i}}^T + \ddot{\theta}{f_{r_i}}^T\right) + \left(K_i - \dot{\theta}^2 M_i\right)\delta_i + C_i\dot{\delta}_i = 0 \tag{29}$$

From the results of discretization using Lagrange's equation, the equation of translational and rotational motions for the *i*-element is obtain

$$2M_i\ddot{\delta}_i + C_i\dot{\delta}_i + \left(K_i - \dot{\theta}^2 M_i\right)\delta_i = \left(\ddot{y}{f_{t_i}}^T + \ddot{\theta}{f_{r_i}}^T\right) \tag{30}$$

Finally, the equation of translational and rotational motions of the system can be written

$$2M_n\ddot{\delta}_n + C_n\dot{\delta}_n + \left(K_n - \dot{\theta}^2 M_n\right)\delta_n = \left(\ddot{y}{f_{t_n}}^T + \ddot{\theta}{f_{r_n}}^T\right) \tag{31}$$

3. COMPUTATIONAL MODEL

Fig. 4 shows the physical model of the SLFM. The SLFM consists of an aluminum beam, a track of the link, a clamp-part, a servo motor to rotate of the link, and a DC motor to make translational motion using ball screw mechanism. The link including the clamp-part where is more rigid than link. Furthermore, at the end of the link, an accelerometer is installed to detect vibration that occur of the system.

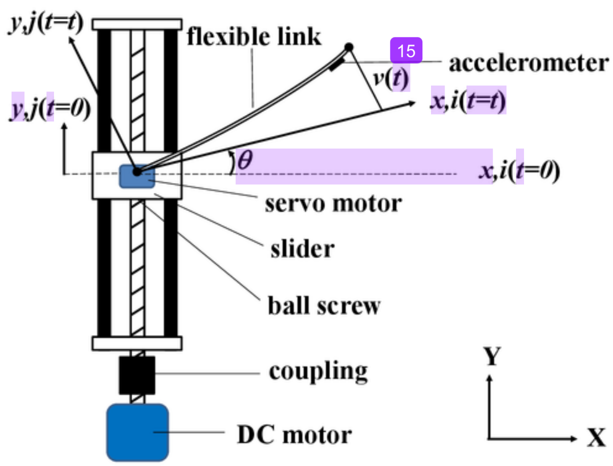


Fig. 4. Physical model of the SLFM

Table 1 shows the physical parameters of the SLFM as the value used to simulate free vibration, translational motion, and translational and rotational motions, in order to obtain an overview of FFT and THR on the system.

TABLE 1: Physical parameters of the SLFM

Property	Symbol	Value
Total length (m)	L	3.30×10^{-1}
Length of the link (m)	l_i	3.00×10^{-1}
breadth of cross section (m)	b_i	2.50×10^{-2}
height of cross section (m)	h_i	1.00×10^{-3}
Cross-section area of the link (m^2)	S_i	1.95×10^{-5}
Cross-section area moment of inertia around i-axis of the link (m^4)	I_i	2.75×10^{-12}
Young's Modulus of the link (N/m^2)	E_i	7.00×10^{10}
The mass density of the link (kg/m^3)	ρ_i	2.70×10^{3}
The damping factor of the link	α	0.10×10^{-3}

4. Result and Discussion

4.1. Time history response of free vibration

Fig. 5 shows time history response of lateral deformation *Vp* on free vibration. The lateral deformation was simulated by using impulse force as an external force. The computational code has been generated at nodal point six on the THR of the SLFM system. The minimum deviation is at the value -0.009 [m] and the maximum deviation is 0.0078 [m].

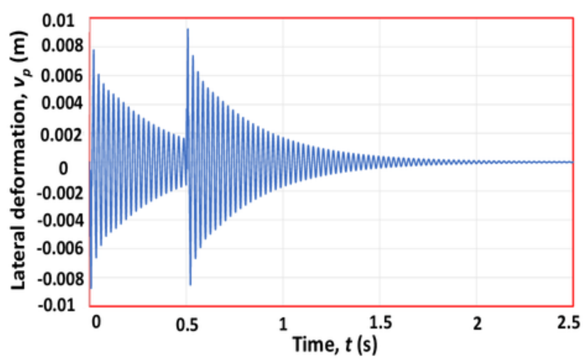


Fig. 5. THR on free vibration of the SLFM

4.2. Fast Fourier Transform Processing

The results obtained from simulation with free vibration on the system, then transferred by the FFT process to obtain the value of natural frequency to the magnitude. Fig. 6 shows calculate the natural frequency of the SLFM system. The result shows that the natural frequency of the system is 8.3 [Hz]. When compared using the calculation method, the natural frequency of the system obtained is 9.1 [Hz], so it can be concluded that the simulation results can be validated correctly.

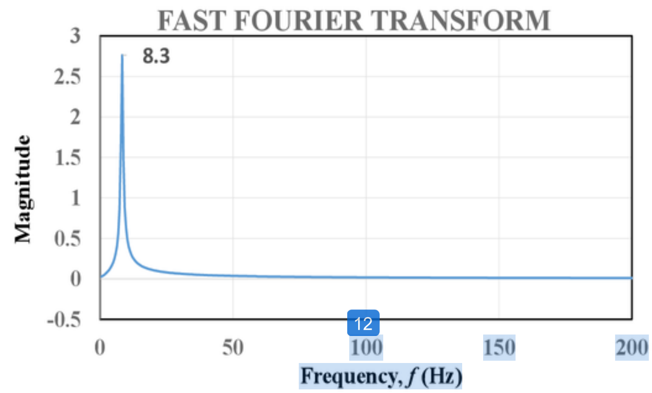


Fig. 6. Natural frequency of the SLFM

4.3. Time history response with excitation force on translational motion

Fig. 7 shows the THR of lateral deformation with excitation force of the SLFM system on translational motion. The computational code has been generated at nodal point 6 on the THR of the SLFM. Lateral deformation was simulated by using excitation force. It can be seen that the lateral deformation get the maximum value at t = 0 [s] and t = 0.5 [s] duo to the existence of excitation force at the time.

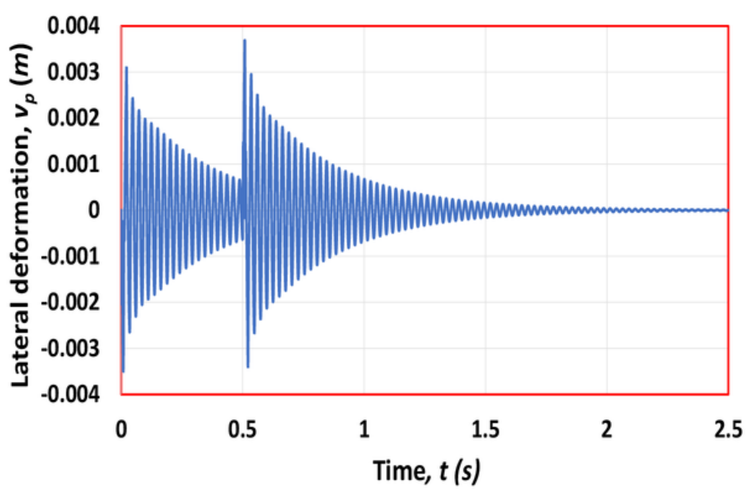
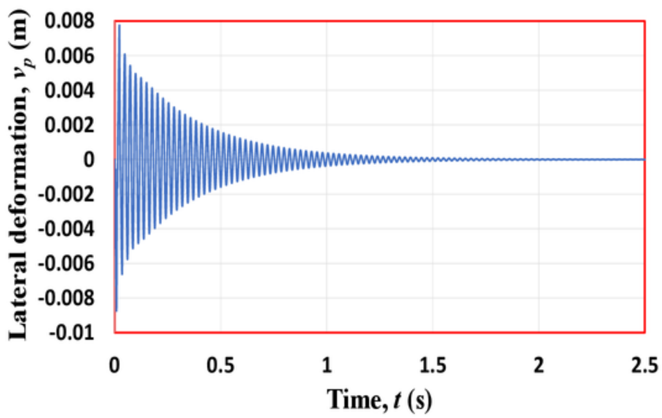


Fig. 7. THR on the system with excitation force experienced translational motion

4.4. Time history response with excitation force on translational and rotational motions.

The simulation results of translational and rotational motions obtain THR for lateral deformation using excitation forces on the system shown in Fig. 8. The Lateral deformation has a greater value compared to translational motion caused by the presence of two kinetic energies that occur namely translational kinetic energy and rotational kinetic energy that takes place simultaneously on the SLFM system. It can be seen that the lateral deformation get the maximum value at t = 0.5 [s] duo to the existence of excitation force at the time. Moreover, Lateral displacement in this condition is smaller when



compared to translational motion. This is due to the presence of two kinetic energies that occur in the system, namely translational and rotational kinetic energy.

Fig. 8. THR on the system with excitation force experienced translational and rotational motions.

5. CONCLUSION

The results of this study have resulted in dynamic modeling of the single-link flexible manipulator robot. The equation of motion for the single-link flexible manipulator that moves translation and rotational has been formulated with the finite element method. The computational code has been successfully developed for use in system dynamics simulations. Simulation and calculation results are presented in the order of time history response and natural frequency with the FFT processing.

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