

Error Performance Analysis of Cooperative Relaying Communications with Fixed Gain and CSI-Assisted Relays

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Abstract – This paper presents a performance closed-form error analysis of cooperative relaying communications with fixed gain and CSI-assisted relays. We derive analytical expressions for the performance of cooperative transmission between a source terminal and a destination terminal using a relaying model. More specifically, we first evaluate the statistical behavior of two multipath fading channels: Rayleigh fading and Rician fading and study the probability density functions of the fading amplitude and the instantaneous signal-to-noise ratio (SNR) per symbol of the channel. Next, we present both simulation and theoretical bit error rate performance of two relaying methods: Amplify-and-Forward (AF) and Decode-and-Forward (DF).

I. Introduction

Recently, due to the high demand data rates in wireless multi-media applications, wireless relaying technologies have received significant research interest [1]. Using information theoretic analysis, researchers have so far shown that the implementation of the cooperative relaying networks in wireless communication system can be an effective way for improving the capacity to convey information from a source terminal to a destination terminal in the network [2]. In the past, research on relaying networks has traditionally focused on studying the performance analysis of dual-hop relayed transmissions with fixed gain relays over different fading scenarios such as Rayleigh [3]-[4] Nakagami- m [5] and lognormal fading channels [6].

A. Cooperative Wireless Relaying Communications

The idea of cooperative wireless relaying communication is based on the cooperatively transmission of a source terminal and a destination terminal by a relaying model. The information from the source is transmitted to the relay before arriving at the destination. Fig 1 shows a graphical representation of such a cooperative wireless relaying communication scheme.

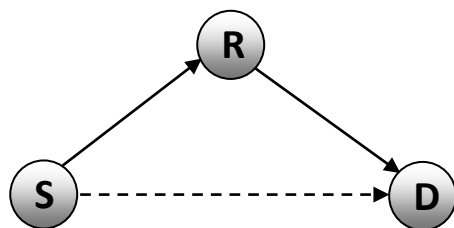


Fig 1 Cooperative wireless relaying communication scheme

In the figure, the three-node network consist of S (Source terminal), R (relay terminal), and D (destination terminal). $S - D$ link is called direct path and $S - R - D$ link is called relay part. If the source terminal has no direct part to the destination terminal due of large distance or lots of fading for instance, it cannot reach directly to send a message. Therefore, the source terminal can forward the message via several intermediate nodes. Here, relay terminal will play a central role in the network. The source terminal broadcasts the signal to both the relay and destination terminals. The relay, then, retransmits the information to the destination. Therefore, the source and the relay cooperate to transmit the signal information simultaneously to the destination.

Next, we look at the basic relaying methods based on which the cooperative wireless relaying communication system is studied.

B. Relaying Methods

Two basic relaying methods commonly used for cooperation are discussed below; Amplify-and-Forward (AF) [5], [7] and Decode-and-Forward (DF) [3], [8].

A simplified demonstration and comparison of these methods is shown in Fig 2.

1) *Amplify-and-Forward (AF)*: Each cooperating node receives a noisy version of the transmitted signal. Then, the node amplifies and retransmits this noisy version. In other words, the relay simply sends forward the signal in the second channel of the source by amplifying the received signal at the relay node and then retransmits them to the destination.

This scheme has some advantages such as better diversity, low complexity or more flexible to synchronization constraints, and no need feedback requirement. However, it has disadvantages such as relay also amplifies the inter-user channel noise and destination receives the simple repetition coding.

2) *Decode-and-Forward (DF)*: Amplifying is not required at the relay node. A cooperating node first decodes the signal transmitted from a source and then retransmits them to the destination. When the transmission from the source is received at the relay node which has enough computing power, the relay completely decodes the signal, re-encodes it with the same codebook used in the original source node's

transmission and then transmits the signal in the second channel of the source node to the destination.

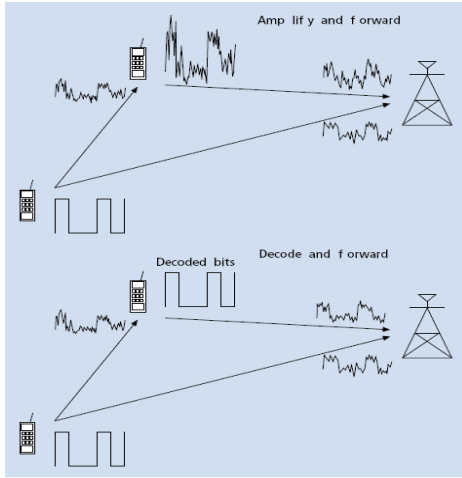


Fig 2 AF and DF relaying schemes (modified from [9])

Although this scheme has the advantages such as reducing the impact of receiving noise, forwarding the exact source information, and applying various coding schemes, as we will see, it also has disadvantages such as fading effects, error propagation in improperly decoded and requiring feedback between Source – Relay terminals. Therefore this scheme has very high complexity.

In this paper, we study the performance of cooperative relaying communications related to wireless communication systems. We discuss the wireless fading channel properties including the modeling of the multipath fading phenomena. Moreover Rayleigh and Rician fading are considered. Next, it is dedicated to the analysis of Amplify-and-Forward and Decode-and-Forward schemes.

II. Modeling of Multipath Fading Channel

The simulation of the statistical behavior of two modeling of multipath fading channels, namely Rayleigh fading and Rician fading, is discussed. We performed the simulation using; $p_\alpha(\alpha)$, probability density function (PDF) of fading amplitude α and $p_\gamma(\gamma)$, the PDF of the instantaneous signal-to-noise power ratio (SNR) per symbol of the channel.

In [10] describes that when fading affects narrowband systems at the receiver, the transmitted signal is modulated by the fading amplitude α and $p_\alpha(\alpha)$, which is dependent on the nature of the radio propagation environment. The transmitted signal is also added with additive white Gaussian noise (AWGN), which is typically assumed to be statically independent of the fading amplitude α and which is characterized by a one-sided power spectral density (PSD) N_o

(W/Hz). Equivalently, the received instantaneous signal power is modulated by α^2 . It follows that we can define the PDF of the instantaneous SNR per symbol of the channel γ , where $\gamma = \alpha^2 \frac{\epsilon_i}{N_o}$ and the average SNR per symbol by $\bar{\gamma} = \Omega \frac{\epsilon_i}{N_o}$, where $\Omega = \overline{\alpha^2}$ is the mean-square value and ϵ_i is the energy per symbol on each node ($i = 1, 2$). Therefore, the PDF of $p_\gamma(\gamma)$ is obtained by the expression [10, Eq. (2.3)]

$$p_\gamma(\gamma) = \frac{p_\alpha\left(\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}\right)}{2\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}} \quad (1)$$

A. Rayleigh Distribution

The Rayleigh distribution is commonly used to describe the statistical delaying time of the received envelope of a flat fading channel, or the envelope of an individual multipath component [11]. In addition, the Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path. The PDF of the channel fading amplitude α which is distributed according to [10, Eq. (2.6)]

$$p_\alpha(\alpha) = \frac{2\alpha}{\Omega} \exp\left(-\frac{\alpha^2}{\Omega}\right), \quad \alpha \geq 0 \quad (2)$$

Fig 3 below shows the simulation result based on (2). In this case, the plot has been generated using 10000 random variable (RV) realizations of the fading amplitude α . We have fixed the total average channel SNR at 10 dB.

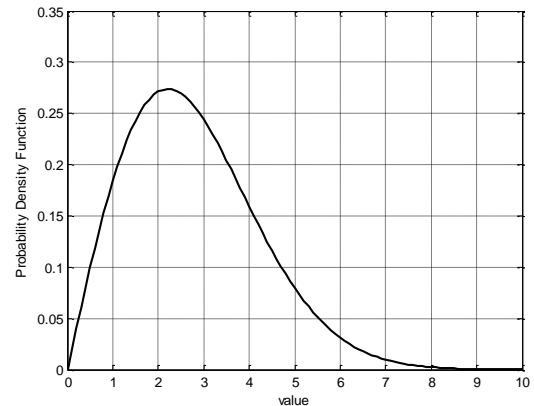


Fig 3 PDF of the Rayleigh distributed channel fading

Moreover, using [10, Eq. (2.3)], the PDF of $p_\gamma(\gamma)$ can be expressed as

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0 \quad (3)$$

Fig 4 below plots the theoretical PDF of the instantaneous SNR per symbol γ . We fix the average SNR at 5, 10 and 20 dB respectively.

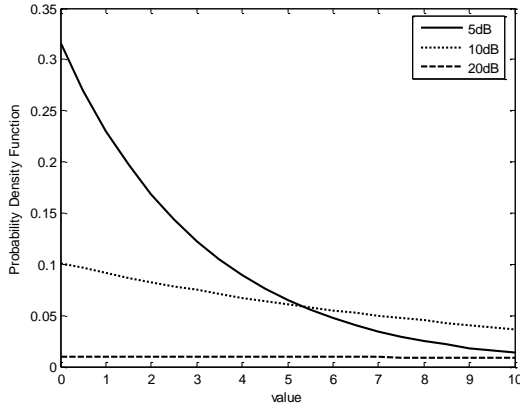


Fig 4 PDF of Rayleigh distributed instantaneous SNR per symbol

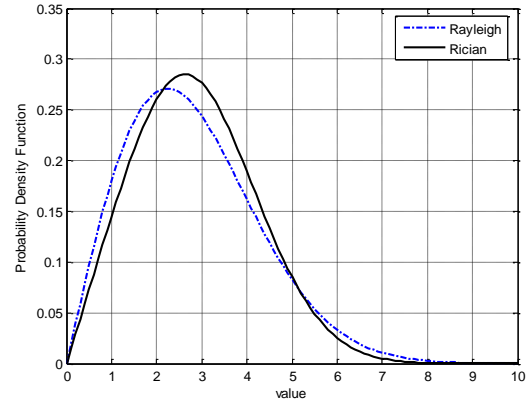


Fig 5 PDF of the Rician distributed channel fading amplitude

B. Rician Distribution

This model is often used to model propagation path consisting one strong direct LOS component and many random weaker components. Rician fading distribution model is also commonly called Nakagami- n model [10]. The Rician distribution is more general than Rayleigh distribution. By setting the LOS component of the Rician distribution, we obtain the Rayleigh distribution.

The Rician distribution is often described in terms on the Rician K -factor. It is defined as the ratio between the signal powers in LOS component over the scattered power (from the indirect paths).

In [12], it is mentioned that K is usually expressed in decibels (dB) and when K (dB) = $-\infty$ it becomes the Rayleigh distribution. As $K \rightarrow \infty$ the Rician distribution approximates that of an AWGN (no fading) channel. Values of the K -factor in indoor/outdoor land mobile applications normally range from 0 – 12 dB [13].

The PDF of the fading amplitude α , can be distributed according to [10, Eq. (2.15)]:

$$p_{\alpha}(\alpha) = \frac{2(1+n^2)\varepsilon^{-n^2}\alpha}{\Omega} \exp\left[-\frac{(1+n^2)\alpha^2}{\Omega}\right] I_0\left[2n\alpha\sqrt{\frac{(1+n^2)}{\Omega}}\right] \quad (4)$$

where $I_0(\cdot)$ is zeroth-order modified Bessel function of the first kind [12] and n is the Nakagami- n fading parameter which ranges from 0 to ∞ and which is related to the Rician K -factor by $K = n^2$.

Fig 5 presents the PDF of the Rician distributed channel fading amplitude α . It also compares the PDFs of the Rician and Rayleigh models. The Rician K -factor is set to be 1 dB and other parameters used are similar when we plot the Rayleigh model.

Moreover, in Fig 6, $p_{\gamma}(\gamma)$ is plotted. The PDF Rician faded the SNR per symbol of the channel, γ , is distributed according to a non-central χ -square distribution and is given by [10, Eq. (2.16)]

$$p_{\gamma}(\gamma) = \frac{(1+n^2)\varepsilon^{-n^2}}{\Upsilon} \exp\left[-\frac{(1+n^2)\gamma}{\Upsilon}\right] I_0\left[2n\sqrt{\frac{(1+n^2)\gamma}{\Upsilon}}\right] \quad (5)$$

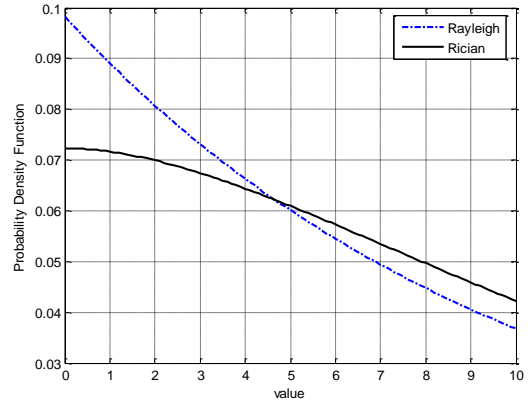


Fig 6 PDF of Rician distributed instantaneous SNR per symbol

III. Error performance Analysis of AF and DF Schemes and Numerical results

We assume that the source terminal is transmitting a signal $s(t)$ during the first time slot with an average power of ε_i , where $i = 1, 2$ is the per hop SNR. The data is a random bipolar bit sequence which is modulated with binary phase shift keying (BPSK). Then, the received signal at the relay terminal, $r_R(t)$, can be expressed as (6). Next, in the second time slot, the relay multiplies the $r_R(t)$ by a gain factor G and then retransmits it to the destination. The received signal at the destination terminal, $r_D(t)$, can be expressed as shown in (7). Note that for both the received signal expressions, the received signal is also added with the AWGN with power N_o [3]-[4].

$$r_R(t) = \alpha_1 s(t) + n_1(t), \quad (6)$$

$$r_D(t) = \alpha_2 G r_R(t) + n_2(t), \quad (7)$$

$$r_D(t) = \alpha_2 G (\alpha_1 s(t) + n_1(t)) + n_2(t), \quad (8)$$

where:

$r_R(t)$: Received signal at R .

$r_D(t)$: Received signal at D via R .

α_1 : Fading amplitude of the channel between S and R .

α_2 : Fading amplitude of the channel between R and D .
 $n_1(t)$: The AWGN signal with power N_{o1} at the input of R .
 $n_2(t)$: The AWGN signal with power N_{o2} at the input of D .
 G : Gain of the relay at R .

The end-to-end SNR at the destination can be written as [4, Eq. (3)]

$$\gamma_{eq} = \frac{\frac{\varepsilon_1 \alpha_1^2}{N_{o1}} \frac{\alpha_2^2}{N_{o2}}}{\frac{\alpha_2^2}{N_{o2}} + \frac{1}{G^2 N_{o2}}}, \quad (9)$$

where:

γ_{eq} : SNR at D
 ε_1 : Power of the transmitted signal at S , where the signal power of $s(t)$ can be written as:

$$E[|s(t)|^2] = \varepsilon_1 \quad (10)$$

 ε_2 : Power of the transmitted signal at R .

Moreover, selecting a gain G in an AF relay system can be classified into two ways; channel state information (CSI)-assisted relay [3]-[4] and fixed gain (blind) relay [3]-[5]. In terms of the CSI-assisted relay, the relay uses the instantaneous CSI from the previous hop (S-R link) and instantaneously power control of the retransmitted signal. This system has been extensively studied in [3]-[4], [7]. On the other hand, fixed gain relays, utilize a fixed value as the gain. Thus a variable instantaneous signal power at the relay output is produced; however on the average the transmitted power can be controlled. This system has been extensively studied in the works of [4]-[5]. A fixed-gain relay, compared to CSI-assisted relay has a lower complexity because it does not want to estimate the source-to-relay link channel.

It is clear from (9) that the gain G defines the equivalent SNR of γ_{eq} between $S - R$ and $R - D$ links. Therefore, the relay gain with the instantaneous CSI-assisted system can be written as [4, Eq. (4)]

$$G^2 = \frac{\varepsilon_2}{\varepsilon_1 \alpha_1^2 + N_{o1}} \quad (11)$$

and the gain of the fixed gain relay system at terminal R can be written as [3, Eq. (12)]

$$G^2 = \frac{1}{\alpha_1^2 + N_{o1}}. \quad (12)$$

A. Instantaneous End-to-end SNR

1) CSI-assisted Relay

Let us first consider CSI-assisted relaying. By substituting (11) in (9) leads to γ_{eq1} , the instantaneous end-to-end SNR at terminal D , γ_{eq1} , can be derived as

$$\gamma_{eq1} = \frac{\alpha_1^2 \alpha_2^2 \varepsilon_1 G^2}{\alpha_2^2 N_{o2} G^2 + N_{o2}} = \frac{\frac{\alpha_1^2 \varepsilon_1}{N_{o1}} \frac{\alpha_2^2 \varepsilon_2}{N_{o2}}}{\frac{\alpha_1^2 \varepsilon_1}{N_{o1}} + \frac{\alpha_2^2 \varepsilon_2}{N_{o2}} + 1}, \quad (13)$$

let $\gamma_1 = \frac{\alpha_1^2 \varepsilon_1}{N_{o1}}$ and $\gamma_2 = \frac{\alpha_2^2 \varepsilon_2}{N_{o2}}$ (the instantaneous of the $S - R$ and $R - D$ links respectively), as a result, (13) can be expressed as [4, Eq. (5)]

$$\gamma_{eq1} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \quad (14)$$

2) Fixed-gain Relay

After that, let us consider the fixed-gain relaying. Let $C = 1 + \frac{\varepsilon_1}{N_{o1}}$ (a constant for a fixed G), and the $S - R - D$ links are assumed to be subject to independent. As a result, γ_1 and γ_2 are exponentially distributed with parameters $\bar{\gamma}_1 = \Omega_1 \frac{\varepsilon_1}{N_{o1}}$ and $\bar{\gamma}_2 = \Omega_2 \frac{\varepsilon_2}{N_{o2}}$ respectively, where $\Omega_i = \overline{\alpha_i^2}$ ($i = 1, 2$) is the average fading power on the i th link. Therefore, by substituting $C, \bar{\gamma}_1$, and $\bar{\gamma}_2$ in (13), the instantaneous end-to-end SNR at terminal D , γ_{eq2} , can be expressed in the case of fixed-gain relay as [4, Eq. (6)]

$$\gamma_{eq2} = \frac{\gamma_1 \gamma_2}{C + \gamma_2} \quad (15)$$

In addition, the theoretical expression of the PDF of the end-to-end SNR in case of fixed gain relaying can be found in [4, Eq. (10)] and is given by

$$p_\gamma(\gamma) = \frac{2}{\bar{\gamma}_1} \exp\left(\frac{-\gamma}{\bar{\gamma}_1}\right) \left[\sqrt{\frac{C\bar{\gamma}_1}{\gamma_1 \bar{\gamma}_2}} K_1\left(2\sqrt{\frac{C\bar{\gamma}_1}{\gamma_1 \bar{\gamma}_2}}\right) + \frac{C}{\bar{\gamma}_2} K_0\left(2\sqrt{\frac{C\bar{\gamma}_1}{\gamma_1 \bar{\gamma}_2}}\right) \right], \quad (16)$$

where $K_0(\cdot)$ is zeroth-order modified Bessel function of the first kind [4]. Note that in Matlab, $K_0(\cdot)$ can be evaluated using the ‘‘Besselk’’ command.

Next, considering Rayleigh fading, we will investigate the PDF of the instantaneous SNR. We have performed computer simulations of (15) and the results are shown in Fig 7. In our simulation, we set 500000 RV realizations of the fading amplitude α . We have fixed $C = 1$ and the total average channel SNRs of both $\bar{\gamma}_1$ and $\bar{\gamma}_2$ at 10 dB.

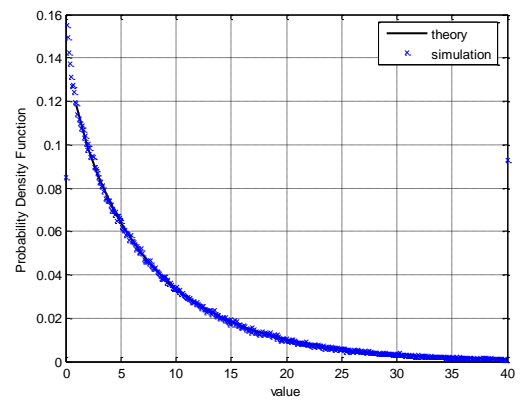


Fig 7 Comparison between the analytical result for the PDF in (15) and the Monte Carlo simulation

B. Error performance

In this section, we present the error performance of both AF and DF relaying schemes. In the previous literature, one of the measures adopted for evaluating the performance of a wireless system is the bit or symbol error rate. In the next sections, we will present both simulation and theoretical bit error rate (BER) of AF and DF relay systems.

1) Amplify-and-Forward Scheme

As we can see from (8) that the received signal at the relay is amplified before forwarding to the destination.

The BER performance with five different values of G for AF relaying scheme is shown in Fig 8. The ratio between the average SNRs of the $S - R$ and $R - D$ links, i.e., $\mu = \frac{\gamma_1}{\gamma_2}$ is equal to 1. In our simulations, we set the average SNR of γ_1 ranges from 0 to 12 dB.

So far, our analysis shows that the BER performance improves as the gain G increases. This is intuitive since, a larger gain corresponds to a high transmit power at the relay.

On the other hand, the Fig 9 shows the BER performance of AF scheme when using different variable of μ . Again, it is clear from the figure that the BER performance improves as μ increases.

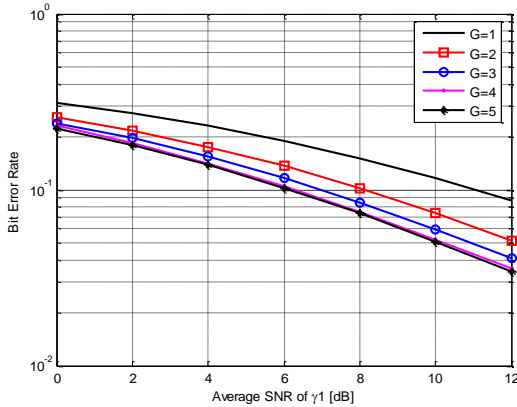


Fig 8 BER performance of the AF scheme with different Gains

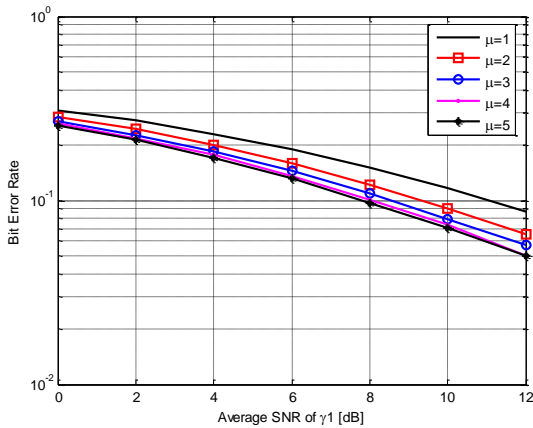


Fig 9 BER performance of the AF scheme with different values of μ

Theoretical Error Probability Analysis

In [5], the average BER is defining by determining the PDF of γ_{zq} and then the averaging the conditional BER in AWGN, $P_b(\epsilon|\gamma)$, over this PDF. Therefore, in mathematically, the expression of $P_b(\epsilon)$ can be written as

$$P_b(\epsilon) = \int_0^\infty p(\epsilon|\gamma) p_{\gamma_{zq}}(\gamma) d\gamma \tag{17}$$

where $p(\epsilon|\gamma) = Q(\sqrt{2\gamma})$ for BPSK and $p_{\gamma_{zq}}$ is the expression given in (16). $Q(\sqrt{2\gamma})$ is the Gaussian Q -function defined as [5], [10, Eq. (4.1)]

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt \tag{18}$$

Moreover, in the special case of Rayleigh fading, the final expression of the theoretical probability of error $P_b(\epsilon)$ in (17) has been evaluated in [5, Eq. (12)] and is given by

$$P_b(\epsilon) = \frac{1}{2} \left(1 - \frac{l}{\sqrt{1+(\frac{l}{\delta_1 \delta_2})^2}} e^l [K_1(l) - K_0(l)] \right), \tag{19}$$

where $l = \frac{C}{(2 + \delta_1 \beta) \delta_2}$ and δ_i represents for $\frac{\epsilon_1}{N_0}$ and $\frac{\epsilon_2}{N_0}$.

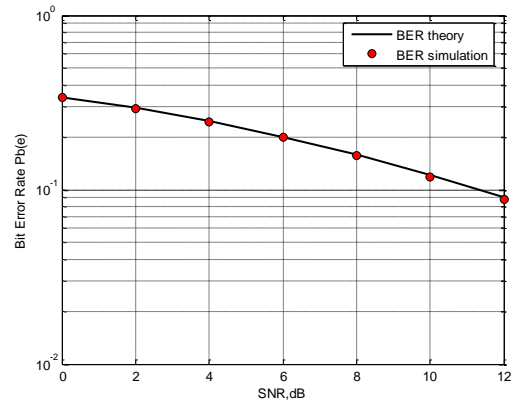


Fig 10 Theoretical and simulated BER performance of the AF scheme

As double check, in Fig 10, we have plotted the theoretical BER in (19) and a simulation result of AF scheme. These results match exactly.

2) Decode-and-Forward Scheme

In this following, we assume that the relay decodes the original message completely and there is no error correcting code added to the data. So there is no chance to correct any wrongly detected bits at the relay. At the relay, every decoded symbol is re-encoded and retransmitted to the destination terminal.

Fig 11 shows the BER performance of DF scheme. The plot has been generated using $G = 1$ and $\mu = 1$. The relay detects the transmitted signal symbol by symbol. In the case of BPSK modulated signal, the symbol/bit is detected as $s(t)$ is $+1$ for $Re[s(t)] \geq 0$ and -1 for $Re[s(t)] < 0$.

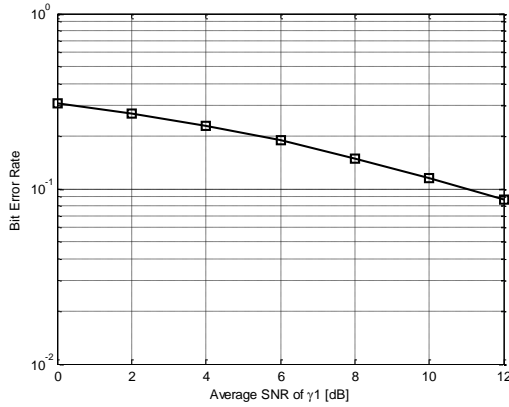


Fig 11 BER performance of the DF scheme

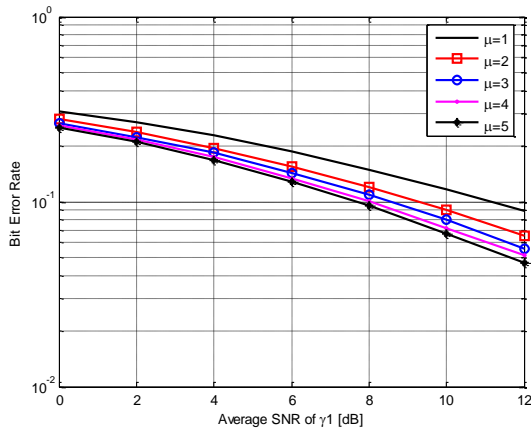


Fig 12 BER performance of the DF scheme with different values of μ

In addition, we examine the BER performance of this scheme with different variable values of μ . As shown in Fig 12, it shows that the BER performance improves as the variable μ increases.

• **Theoretical Error Probability Analysis**

In [3], in the case relay path of DF scheme, the relay decodes the transmitted signal and then retransmits the detected version to the destination. Therefore, the transmitted signal undergoes two states of decoding is cascade, and the average BER is given by [3, Eq. (25)]

$$P_b(e|\gamma_1, \gamma_2) = P_b(e|\gamma_1) + P_b(e|\gamma_2) - 2P_b(e|\gamma_1)P_b(e|\gamma_2) \tag{20}$$

Note that $P_b(e|\gamma_1)$ and $P_b(e|\gamma_2)$ are the average BER of the single *S-R* and *R-D* links respectively. When the averaged over the two independents RVs of γ_1 and γ_2 reduce to [3, Eq. (26)]

$$P_b(e) = P_b(e_1) + P_b(e_2) - 2P_b(e_1)P_b(e_2) \tag{21}$$

It is well known that for a Rayleigh faded channel, the BER of BPSK is given by [5]

$$P_b(e_1) = \int_0^\infty Q(\sqrt{2\gamma}) p_{\gamma_{rq}}(\gamma) d\gamma \tag{22}$$

Therefore substituting (22) in (21) we finally get the average BER for the DF system as [14]

$$P_b(e) = \frac{1}{2} \left(2 - \left(1 - \sqrt{\frac{\gamma_1}{1+\gamma_1}} \right) - \left(1 - \sqrt{\frac{\gamma_2}{1+\gamma_2}} \right) - \left(\left(1 - \sqrt{\frac{\gamma_1}{1+\gamma_1}} \right) \left(1 - \sqrt{\frac{\gamma_2}{1+\gamma_2}} \right) \right) \right), \tag{23}$$

Furthermore, Fig 13 plots the simulated BER performance of the DF scheme validating the accuracy of our mathematical approach.

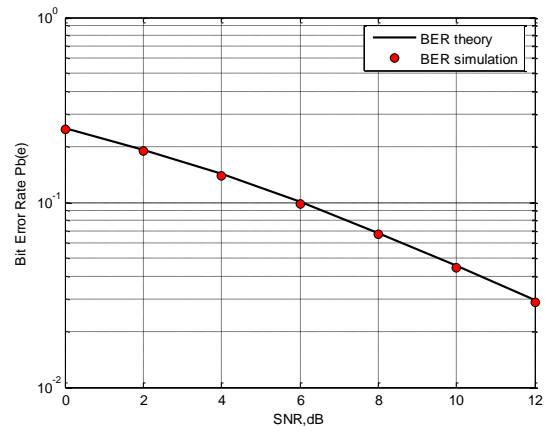


Fig 13 Theoretical and simulated BER performance of the DF scheme

IV. Conclusions

We presented the behavior of multipath fading channel by simulating and comparing the performance of Rayleigh and Rician distributions. It was found that the Rayleigh fading is the most reliable model when there is no dominant propagation along LOS between the transmitter and receiver. Therefore, if there is a dominant LOS, the Rician fading may be more applicable. We explored the performance of the cooperative relaying network with AF and DF schemes respectively. It was shown that the AF scheme can achieve better performance depending on the choice of the relay gain. This analysis is useful to investigate the performance of AF and DF relaying schemes subject to different fading condition for both source-relay and relay-destination links.

V. References

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