Joint Odd-Even Quantisation in Cartesian Delta-Sigma (ΔΣ) Upconverters

Sirmayanti Sirmayanti[1,3], Vandana Bassoo[2] and Mike Faulkner[1]
[1] College of Engineering and Science, Victoria University, Australia
[2] School of Innovative Technologies and Engineering, University of Technology-Mauritius, La Tour Koenig, Pointe-aux-Sables, Mauritius
[3] The State Polytechnic of Ujung Pandang, Makassar, Indonesia
E-mail: sirmayanti.sirmayanti@live.vu.edu.au

***Abstract*—Delta-Sigma (ΔΣ) converters enable good in-band signal to quantisation noise performance from coarse quantisers. They achieve this by oversampling the signal and noise shaping the quantisation noise such that it does not fall in the band of interest. We apply ΔΣ techniques to the direct generation of RF modulated signals using two (binary) or three level waveforms, which are pulse width or pulse position modulated. The ΔΣ filters operate on the baseband IQ signals and quantisation is in the polar domain. Two-dimensional quantisation is required and low complexity methods of achieving this are described and compared. The complexity is more than halved if pulse widths are restricted to an even or odd number of clock cycles. This comes at a 5 dB loss in RF signal to noise performance.**

I. Introduction

ΔΣ techniques are most well known for their use in AD and DA converter structures. These schemes are almost entirely based on the conversion of low-pass signals. The best know early application of ΔΣ DACs was for the CD player. Here the sample rate was increased to reduce the (quantisation) noise power spectral density and then shaped by a first order network to further reduce the noise in the lower frequencies. In this way high fidelity signal reproduction was possible from DAC's of reduced resolution. Fig 1 shows a block diagram of a second order ΔΣ converter. The scheme is so effective that DAC (quantiser) resolutions of 1-bit are possible if the oversampling rate is high enough [1].

Recent research is now applying these techniques to band-pass signals such as found in radio frequency transceivers [2]. There are number of challenges to developing such schemes. First, the carrier frequency is of the same order as the sample rate, secondly the bandwidth, error vector magnitude (EVM) and spectral mask of any transmitted RF signal must be met, and thirdly any design must be realisable in today's silicon technology. These three factors are all inter-related and form a complex trade-off between performance, complexity and energy consumption. We will concentrate on all digital up-converter structures. The binary nature of their output removes the need for analog components in the up-conversion chain. Their outputs can be used directly to drive switched mode power amplifiers (SMPA) (Class S or D) for high efficiency operation [3]. There are three major approaches; bandpass ΔΣ with carrier frequency, $f_c = f/4$ [3]; Cartesian lowpass ΔΣs with quadrature up-conversion to $f_c = f/4$ [4]; and Cartesian filtered polar quantised conversion with $f_c = \frac{f}{(4*2^p)}$ where the integer p $>= 0$ [5]. The latter is the topic of this paper since the factor p allows a trade-off between the RF oversampling factor OSR_{RF} and the fidelity of the quantiser. The polar quantisation process requires complex two-dimentional(2-D) processing in the IQ plane to identify the quantisation point closest to the desired signal. The 2-D quantisation process termed 'joint' quantisation can be simplified into two 1-D processes by choosing a subset of the quantisation points and changing the signals to polar prior to quantisation [6]. Two subsets are possible named "even" and "odd" and will be described in section II as part of the overall architecture. Section III will describe the more optimal 'joint' process and a reduced complexity implementation is described in Section IV.

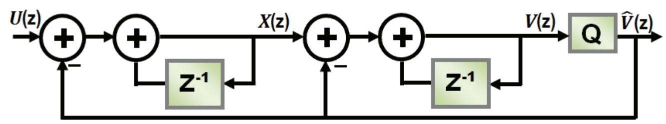

Fig. 1. ΔΣ with a second-order noise shaping filter and quantiser, Q.

II. Cartesian ΔΣ with Polar Quantisation

Polar ΔΣ modulators were introduced by [7] to generate a pulsed square wave waveform with a number of quantised pulse widths and pulse positions to control amplitude and phase respectively. It is important to limit the number of edges in the waveform to two per period of the RF carrier, since each edge represents power loss in any subsequent amplifying stage. The polar ΔΣ did this by converting the digital baseband signals to polar and then separately using ΔΣs to quantise the amplitude and phase components. The problem with the scheme was the high noise floor caused by passing the band-width expanded polar signals through the ΔΣ filters. A partial solution to the problem involved using Cartesian signals in the ΔΣ filters and polar signals in the quantiser. This improved the spectral performance, but at the expense of a rectangular to polar conversions inside the ΔΣ loop [8]. The structure is shown in Fig 2. The quantised amplitude, $\hat{V}_R$, and phase theta,

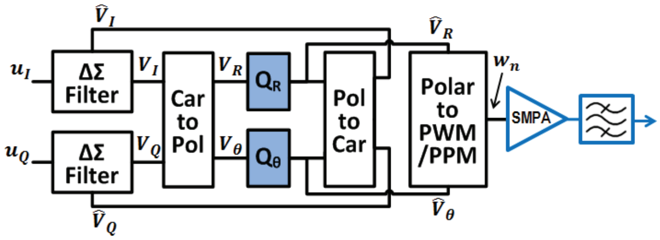

Fig. 2. Cartesian $\Delta\Sigma$ with polar quantisation.

$\hat{V}_{\Theta}$, that are fed back to the $\Delta\Sigma$ in Cartesian format and also fed to the switching power amplifier in PWM/PPM format (see Fig 3. for waveform examples). A bandpass filter at the output of the SMPA reconstructs the sinusoidal RF signal. SMPA's can use binary input signals or tri-state input signals if a full bridge structure is employed [9]. Tri-state will remove the even harmonics.

III. Odd and Even Quantisation

The number of quantisation levels is a function of the $OSR_{RF} = \frac{f_{clock}}{f_c}$, or the number of clock cycles per period of the RF carrier. The amplitude and phase of the switching waveform can be updated in half period increments of the RF carrier giving the sample rate of the $\Delta\Sigma$ filters, $f_s = \frac{2f_c}{\alpha}$ where α is a positive integer. Normally α is greater than one because of the difficulty of making high speed $\Delta\Sigma$ filters.

The quantised amplitude levels are given by the amplitude of the fundamental sinusoidal component of the repeating pulsed waveform. There are $N_A = \frac{OSR_{RF}}{2} + 1$ different possible amplitudes. For the $OSR_{RF} = 8$ case these amplitudes correspond to pulse widths of 0, 1, 2, 3 and 4 clock periods; the latter two waveforms are shown in Fig 3. What is also noticeable in Fig 3 is that a change in carrier amplitude can be accompanied by a change in phase reference of half a clock period or $2*\pi/(2*OSR_{RF})$ radians. This makes quantisation difficult and so previous work has restricted the amplitude quantisation levels to pulse widths that are either an even number of clock periods or an odd number of clock periods because the phase reference does not then change [10]. In the above example, this would give pulse widths of either 0,1,3 or 0,2,4 clock periods. The number of quantised amplitude levels reduces to $N_{A,odd} = N_{A,even} = \frac{OSR_{RF}}{4} + 1$ and there are $N_P = OSR_{RF}$ quantised phases. The total number of quantisation points is therefore $N_{Q,odd} = N_{Q,even} = OSR_{RF}(\frac{OSR_{RF}}{4}) + 1$.

The amplitude and phase quantisation points can be described in the phase plane of the output carrier signal. The V_I and V_Q signal from the $\Delta\Sigma$ filters form the input vector that is be quantised in terms of amplitude and phase. The quantisation points, P^i, are the intersection of the N_A amplitude rings with the N_P phase spokes. The input vector must be quantised to the closest quantisation point. This is shown in Fig 4 for odd (LHS) and for even (RHS) quantisation. In both cases the phase and amplitude decisions are orthogonal, so the selection of amplitude and phase of the closest quantisation point can be independent.

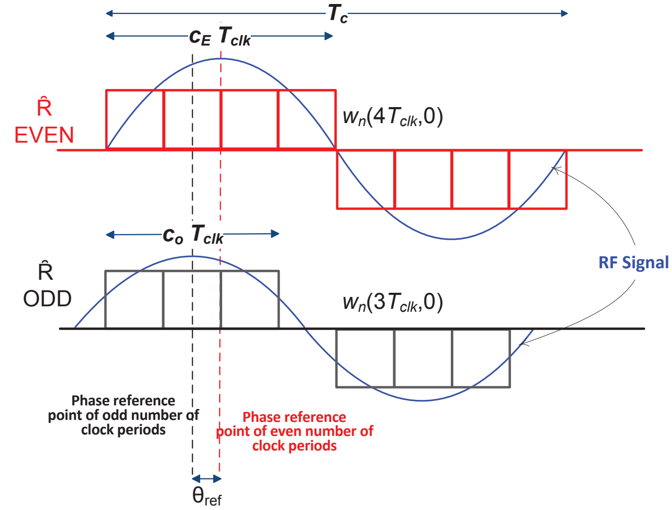

Fig. 3. The PWM/PPM output for $OSR_{RF} = 8$. Note, a two level signal version would set the negative pulses to zero.

IV. Complexity, Sample Rate and Bandwidth

All DSP systems scale with the sample rate and so the higher the sample rate the larger the bandwidth of the noise transfer function null. This is clearly desirable. $\Delta\Sigma$ systems are a feedback system and so the maximum sample rate (or minimum sample period) is set by the time it takes to process all blocks in the feedback loop. This also includes the quantiser. It is therefore important to make any quantisation process fast.

The benefit of the Even or Odd pulse width schemes is that they allow the independent quantisation of phase and amplitude, such that these operations can be performed in parallel, increasing speed and leading to the structure of Fig 2. The downside is that only half the possible quantisation points are used and this increases the quantiser error.

In Fig 2 quantisation is a three stage process, Cartesian to polar conversion, followed by quantisation and then polar to Cartesian conversion. The last stage is trivial because quantisation has occurred and there are only a few states to convert; hence a look-up-table is the best way to go. The first and quantisation stages can be combined together. A possible solution is to use a few iterations of an iterative algorithm such as CORDIC.

The feedback structure of $\Delta\Sigma$ converters makes them particularly tolerant to errors in the quantiser decision regions. This fact can be exploited to reduce the number of iterations in the CORDIC algorithm and so reduce latency. Using pre-computing techniques, it is possible to design a Cartesian $\Delta\Sigma$ with polar quantiser such that it has almost no effect on the maximum sample rate [5].

V. Joint Even-Odd Quantisation

In order to get the benefit of the reduced quantisation noise provided by the full family of quantisation points, we

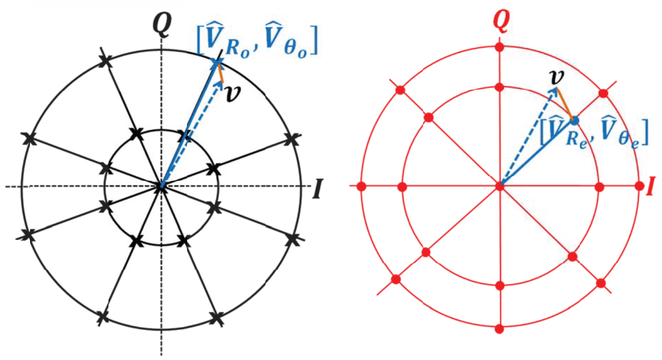

Fig. 4. Quantisation points, P^i, in the IQ plane (or phase plane), $OSR_{RF} = 8$. LHS Odd, RHS Even.

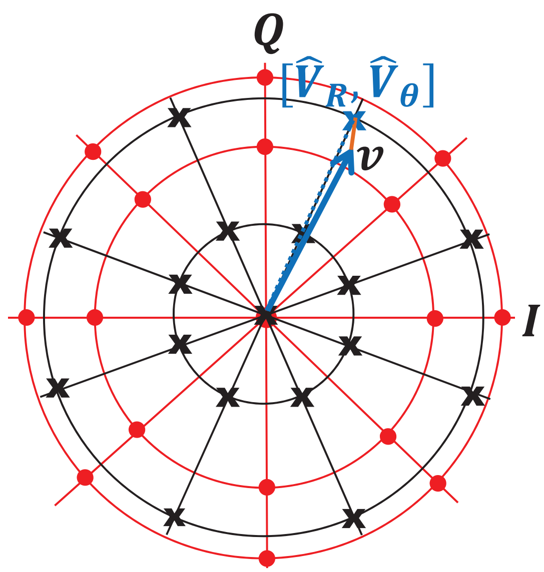

Fig. 5. Phase plane for joint quantisation.

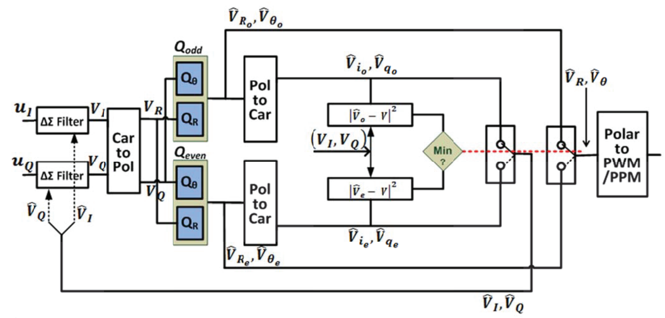

Fig. 6. Proposed reduced complexity quantiser using the combined method.

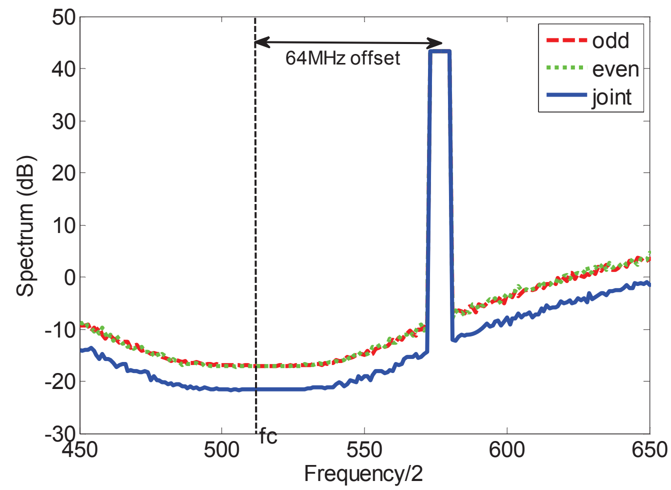

Fig. 7. The simulated output spectrum of an OFDM signal with 10MHz bandwidth. $OSR_{RF} = 32$, with a second-order $\Delta\Sigma$ filter. The carrier is offset from the nominal f_c of 512MHz.

shall now consider implementation options for Joint Even-Odd quantisation shown in Fig 5. Intuitively speaking the quantisation error vector will be approximately half that of the Even or Odd modes considered separately. The number of quantisation points for the Joint scheme is now: $N_{Q,joint} = OSR_{RF} * (\frac{OSR_{RF}}{2}) + 1$.

We wish to choose the quantisation point closest to the input vector (shown in blue). Two methods will be considered.

A. Exhaustive Search

This 2-D search involves calculating the squared distance between the input vector and each quantisation point. The quantisation point, P^{opt}, with the minimum distance is chosen. Five instructions are required to calculate the squared distance; two subtractions two square operations and one summation. Selecting the smallest of the outputs requires $N_{Q,joint} - 1$ compare (subtract) operations. In terms of latency, we note that all the squared distances can be calculated at once using parallel hardware. The latency is dominated by the compare operations.

B. Combined $Odd + Even$ search

In this method the optimum Odd quantisation point is chosen and the optimum Even quantisation point is chosen. These two points are then compared to give the overall optimum choice. The method allows the simpler 1-D searches to be performed, similar to Fig 2, with a final single 2-D squared magnitude comparison. The block diagram is shown in Fig 6.

The Cartesian to Polar operation can be achieved by the Cordic algorithm which approximately doubles the phase accuracy every iteration. If there are OSR_{RF} phases, then the number of Cordic iterations should be ln2(OSR_{RF}) and each iteration has an addition and a compare operation. The amplitude accuracy converges faster and is not a problem. Polar to Cartesian is a look-up-table operation and takes 1 instruction.

VI. Results

Table 1 shows the breakdown of the number of operations for the Odd/Even scheme and both the exhaustive and combinational quantisation schemes for the Joint scheme. The Odd/Even schemes are dominated by the quantisation process, with the polar-cartesian transforms only becoming significant

TABLE I
THE NUMBER OF ARITHMETIC OPERATIONS TO IMPLEMENT THE QUANTISER

Number of Operation						
Scheme	Equation	Q Model	OSR			
			4	8	16	32
Odd (or Even) Quantisation						
Cartesian to Polar	$2ln_2(OSR)+3$	Odd/ Even	7	9	11	13
Quantise Amplitude and Quantise Phase	$(N_p+N_A)=(\frac{5OSR}{4})+1$	Odd/ Even	6	11	21	41
Polar to Cartesian (LUT)	1	Odd/ Even	1	1	1	1
Total	$2ln_2(OSR)+(\frac{5OSR}{4})$+5	Odd/ Even	14	21	33	55
Joint Quantisation, Method-1 : exhaustive search						
Select closest constellation point	$N_{Q,joint}-1$	Joint	8	32	128	512
Minimize $\arg_i min(\Vert R-P^i_{all}\Vert)$ i=1;$N_{Q,joint}$	$5N_{Q,joint}=5(\frac{OSR^2}{2})+1)$	Joint	45	165	645	2565
Total	$6(\frac{OSR^2}{2}+1)-1$	Joint	53	197	773	3077
Joint Quantisation, Method-2 : combination odd+even						
Cartesian to Polar	$2ln_2(OSR)+3$	Odd& Even	7	9	11	13
Quantise Amplitude and Quantise Phase (number includes both Odd and Even)	$2(N_p+N_A)=2(\frac{5OSR}{4}+1)$	Odd& Even	12	22	42	82
Polar to Cartesian (LUT) and selection Odd or Even	$2(1+5)+1=13$	Odd& Even	13	13	13	13
Total	$(2ln_2(OSR)+3)+2(\frac{5*OSR}{4})+1)+13$	Odd& Even	32	44	66	108

at the lowest OSR. For the Joint quantisation scheme the combined method shows an increasing performance benefit over the exhaustive search method as the OSR_{RF} increases. The benefit of Joint quantisation over Odd or Even quantisation is clearly shown in Fig 7 where an across the band 5dB reduction in the noise floor is apparent.

VII. CONCLUSION

$\Delta\Sigma$ systems with 2-D polar quantisation can be used to generate PWM/PPM signals suitable for radio frequency transmission. These give good bandwidth and noise performance when the $\Delta\Sigma$ filtering is performed in the IQ domain rather than the polar domain. This makes quantisation more difficult because Cartesian to polar conversion is required inside the $\Delta\Sigma$ feedback loop. The simplest quantisation occurs when pulse widths are either restricted to either an even number of clock cycles or an odd number of clock cycles. Quantisation can then be done in amplitude and phase separately. Joint quantisation, which uses both odd and even pulse widths gives a 5 dB reduction in noise floor, but the complexity of the quantiser more than doubles. The best quantiser for joint quantisation does the quantisation of the Odd and Even points separately and then chooses the closest to the input signal vector.

REFERENCES

[1] R. Schreier, and G.C. Temes. *Understanding Delta-Sigma Data Converters*, Wiley-IEEE Press, 2004.
[2] R. Schreier and M. Snelgrove, "Bandpass sigma-delta modulation," *Electron Letters*, vol. 25, pp. 1560-1561, Nov. 1989.
[3] S. P. Stapleton, "High efficiency RF Power Amplifiers Using Bandpass Delta-Sigma Modulators," *Agilent Technologies*, Mar. 2003.
[4] M. Helaoui, S. Hatami, R. Negra, and F. Ghannouchi, "A novel architecture of Delta-Sigma Modulator Enabling All-Digital Multiband Multistandard RF Transmitters Design," *IEEE Trans. Circuits and Systems II.*, vol. 55, no.11, pp. 1129-1133, 2008.
[5] V. Bassoo, L. Linton, M. Faulkner, "Analysis of Non-Uniform Polar Quantisers in a Sigma Delta Transmitter Architecture," *IEEE Trans. Circuits and Systems I: Regular Papers* , vol.59, no.12, pp.3060-3070, Dec. 2012.
[6] S. Sirmayanti, V. Bassoo, H. King, and M. Faulkner, "Odd-even quantisation and Cartesian Delta-Sigma (DS) upconverters for transmitter design," in *Proc. IEEE International Conference on Communication Systems (ICCS)*, pp. 100-104, Nov. 2012.
[7] J. Keyzer, R. Uang, Y. Sugiyama, M. Iwamoto, I. Galton, P.M. Asbeck: "Generation of RF Pulsewidth Modulated Microwave Signals using Delta-Sigma Modulation," in *Proc. Microwave Symposium Digest, 2002 IEEE MTT-S International*, vol.1, pp. 397-400, 2002.
[8] V. Bassoo and M. Faulkner, "Sigma-delta digital drive signals for switch-mode power amplifiers," *Electronics Letters*, vol.44, no.22, pp. 1299-1300, Oct. 2008.
[9] V. Bassoo (2010), *A digital up-conversion architecture for future high efficiency wireless base stations*. PhD thesis, Victoria University:Australia.
[10] S. Sirmayanti, V.Bassoo, H.King and M.Faulkner, "OFDM performance in odd-even quantisation in Cartesian Delta Sigma Upconverters," in *Proc. IEEE Signal Processing and Communication Systems (ICSPCS) International Conf.*, pp.1-5, Dec. 2012.