

PAPER • OPEN ACCESS

## The 5<sup>th</sup> International Symposium on Material, Mechatronics and Energy The 5<sup>th</sup> ISMME 2018

To cite this article: 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **619** 011001

View the [article online](#) for updates and enhancements.

You may also like

- [An Investigation to the Effects of Impact Strength on Laminated Notched Composites used in Prosthetic Sockets Manufacturing](#)  
Ehab N. Abbas, Muhanad Al-Waily, Tariq M. Hammza et al.
- [The National Mechatronic Platform. The basis of the educational programs in the knowledge society](#)  
V Maties
- [Classification of parts used in mechatronic products and produced by permanent-mold casting methods](#)  
V Zaharinov, I Malakov, S Nikolov et al.



**ECS** The Electrochemical Society  
Advancing solid state & electrochemical science & technology

243rd Meeting with SOFC-XVIII

Boston, MA • May 28 – June 2, 2023

**Accelerate scientific discovery!**

Learn More & Register

The advertisement features a dark blue background on the left with white and orange text. On the right, there is a photograph of a woman with blonde hair, wearing a black top and a lanyard, smiling and standing behind a podium with a laptop. A blue button with white text 'Learn More & Register' is overlaid on the image.

# The 5<sup>th</sup> International Symposium on Material, Mechatronics and Energy The 5<sup>th</sup> ISMME 2018

Editors:

Dr Rafiuddin Syam Hasanuddin University, Indonesia

Professor Keigo Watanabe Okayama University, Japan

Professor Adi Maimun bin Abdul Malik UTM-Malaysia

Azwar Hayat Hasanuddin University, Indonesia

Dr Andi Amijoyo Mochtar Hasanuddin University, Indonesia

Dr Muh. Zubair Alie Muis Hasanuddin University, Indonesia

Engineering Faculty of Hasanuddin University  
Graduate School of Hasanuddin University  
Research and Community Services Institute of Hasanuddin University

November 2018



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

# Foreword

First, we would like to thank all researcher who are already send the results of scientific research papers and participated in the 5<sup>th</sup> International Symposium on Material, Mechatronics and Energy 2018. All papers in this volume has presented at ISMME 2018 by oral presentation. The papers have been peer reviewed through processes administered by the proceedings Editors. Reviews were conducted by expert referees to the professional and scientific standards expected of a proceedings journal published by IOP Publishing.

Our theme is Challenges and Opportunities of Materials Engineering, Mechatronics and Energy towards independence of independent and sustainable technology products. Themes have been given an important role of Indonesian Development of Industrial Manufacture strategic plan, where the Indonesian people are still in desperate need of technology in these areas, material, mechatronics and energy.

Today Issues is still on Industry 4.0, they are five items should be considered:

1. Scalability; The automation principle of Industry 4.0 could help to facilitate improved scalability among companies in the manufacturing sector.
2. Security; One of the foremost concerns about Industry 4.0 among manufacturers is the possibility of mishaps due to glitches in cognitive computing.
3. Control and Visibility; As manufacturing networks globalize, it is crucial to make digital processes visible to all points of a system. When fully implemented, the principles of Industry 4.0 support responsiveness by making information available worldwide within a fraction of a second.
4. Customer Satisfaction; The process will be fully transparent along all stops on the manufacturing chain, from the moment someone places an order or submits a design until the moment when shipments arrive. Industry 4.0 will facilitate co-creation capabilities between manufacturers and related entities on a global scale.
5. Customization; Industry 4.0 could take customization to new levels with the use of commercial 3-D printers, which there are 23,000 of in use worldwide.

We hope many researchers play on such conditions. Finally, thanks to all of my college in Faculty of Engineering Hasanuddin University, Okayama University, Graduate School of Unhas, Research and Community Services Institute of Unhas and Polytechnik State of Ujung Pandang.

Makassar-Gowa, November , 2018

Yours

Dr. Ir. Muhammad Arsyad Thaha, MT  
Dean of Engineering Faculty of Hasanuddin University

## **Peer Review Statement**

All papers published in this volume of Journal of Physics: Conference Series have been peer reviewed through processes administered by the proceedings Editors. Reviews were conducted by expert referees to the professional and scientific standards expected of a proceedings journal published by IOP Publishing.

## Editors

- Rafiuddin Syam, PhD Hasanuddin University, Indonesia  
Email: rafiuddin@unhas.ac.id
- Professor Keigo Watanabe Okayama University, Japan  
Email: watanabe@sys.okayama-u.ac.jp
- Professor Adi Maimun bin Abdul Malik UTM-Malaysia  
Email: adi@utm.my
- Dr Azwar Hayat Hasanuddin University, Indonesia  
Email: azwar.hayat@gmail.com
- Dr Andi Amijoyo Mochtar Hasanuddin University, Indonesia  
Email: andijoyo@yahoo.com
- Dr Muh. Zubair Alie Muis Hasanuddin University, Indonesia  
Email: bair\_offshore@yahoo.co.id

## Steering Committee

Rafiuddin Syam, PhD - Unhas- Indonesia (Chairman)  
Prof. Keigo Watanabe-Okayama University-Japan  
Prof. Shinfuku Nomura- Ehime University-Japan  
Prof. Dr. Adi Maimun bin Abdul Malik-UTM-Malaysia  
Prof. Satoru Shibata-Ehime University-Japan  
Prof. Mitsuhiro Okayasu-Okayama University-Japan  
Prof. Mitsu Okamura-Ehime University-Japan  
Prof. Kiyotaka Izumi—Saga University-Japan  
Prof. Jackrit Suthakorn, PhD- Mahidol University, Thailand  
Prof. Sherwin Guirnaldo, PhD – Mindanao State University-Philippine  
Prof. Lanka Udawatta, PhD – Moratuwa University - Srilanka  
Prof. Samy F. M. Assal, PhD, Alexandria University- Egypt  
Prof. Bondan T. Sofyan-UI-Indonesia  
Dr. Indrawanto-ITB-Bandung-Indonesia  
Heru Santoso Budi Raharjo-Ir.M.Eng.Dr-UGM-Yogyakarta-Indonesia  
Dr. Gesang Nugroho, ST, MT-UGM-Yogyakarta-Indonesia  
Dr. Kusmono—UGM-Yogyakarta-Indonesia  
Dr.Ir. Yudi Satria Gondokaryono ITB-Bandung-Indonesia  
Dr. Ir. Samiadji Herdjunto, M. Sc – UGM-Yogyakarta -Indonesia  
Ir. Oyas Wahyunggoro, M.T., Ph.D—UGM-Yogyakarta-Indonesia  
Hendro Nurhadi, Dipl. Ing., Ph.D—ITS-Surabaya-Indonesia  
Dr. Syaeful Bakhri -BATAN-Indonesia  
Dr. Abdul Muis, UI-Jakarta-Indonesia  
Prof. Dr.-Ing Nandy Setiadi Djaya Putra-UI-Indonesia  
Prof. Dr.H.Hammada Abbas – Hasanuddin University-Indonesia  
Prof. Effendi Arief– Hasanuddin University-Indonesia  
Prof.Dr. Syamsul Arifin– Hasanuddin University-Indonesia  
Prof. Dr. Ir. Salama Manjang— Hasanuddin University-Indonesia  
Dr. Zulfajri Basri Hasanuddin – Hasanuddin University-Indonesia  
Dr.-Ing Wahyu H Piarah— Hasanuddin University-Indonesia  
Dr. Johannes Leonard – Hasanuddin University-Indonesia  
Dr. Ir. Arsyad Taha, MT – Hasanuddin University-Indonesia

## Organized Committee

Azwar Hayat, ST, M.Sc., Ph.D  
Dr. Kimiko Matonaka, MEng  
Dr. Eng Armin Lawi, MT  
Dr. Jalaluddin, ST,MT  
Dr. Andi Amijoyo, MT  
Dr. Muh. Syahid ST,MT  
Dr. Faizal Arya Samman  
Rahimuddin, ST,MT, PhD  
Dr. Eng Armin Lawi, MT  
Dr. Abdul Kadir Muhammad  
Dr. Meuthia Farida, ST, MT  
Muhammad Zubair Muis Alie, ST,MT, PhD  
Dr. Wiwik Wahida Usman, MT  
Dr. Asniawati Kusno, MT  
Dr. Chaerul Paotonan  
Dr. Adi Tonggiroh, MT  
Dr. H. Ilyas Renreng, MT  
Dr. Abd. Mufti Radja  
Dr. Muh. Syahid, ST, MT  
Dr. Haerul Arsyad, ST, MT  
Dr. Indrabayu  
Dr. Intan Sari Areni, MT  
Dr. Andi Haris Muhammad  
Dr. Ir. Ulfa Ria Irfan, MT  
Dr. Dewiani, MT  
Dr. Hasdinar Umar, ST,MT  
Dr. Sufriadin, ST, MT  
Dr. Nurul Jamala  
Arham Hamid, SE , MSi  
Amiruddin, ST, MT

PAPER • OPEN ACCESS

## Finite element analysis on vibration of a flexible single-link manipulator moved translationally

To cite this article: D Dermawan *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **619** 012050

View the [article online](#) for updates and enhancements.

You may also like

- [Design of a bistable electromagnetic coupling mechanism for underactuated manipulators](#)  
Malaka Miyuranga Kaluarachchi, Jee-Hou Ho, Samer Yahya et al.
- [Hybrid control of a rotational flexible beam using enhanced PD feedback with a nonlinear differentiator and PZT actuators](#)  
Dong Sun, Jinjun Shan, Yuxin Su et al.
- [Collective effects of link failures in linear flow networks](#)  
Franz Kaiser, Julius Strake and Dirk Witthaut



**244<sup>th</sup> Electrochemical Society Meeting**

October 8 – 12, 2023 • Gothenburg, Sweden

50 symposia in electrochemistry & solid state science

▶ **Deadline Extended!**  
**Last chance to submit!**

**New deadline:**  
**April 21**  
**submit your abstract!**



# Finite element analysis on vibration of a flexible single-link manipulator moved translationally

D Dermawan<sup>1,2,\*</sup>, H Abbas<sup>1</sup>, R Syam<sup>1</sup>, Z Djafar<sup>1</sup>, A K Muhammad<sup>2</sup>

<sup>1</sup>Department of Mechanical Engineering, Hasanuddin University, Indonesia 90245

<sup>2</sup>Department of Mechanical Engineering, State Polytechnic of Ujung Pandang, Indonesia 90245

\*E-mail: [dermawan@poliupg.ac.id](mailto:dermawan@poliupg.ac.id)

**Abstract.** The objectives of this study are to formulate the equation of motion of a flexible single-link manipulator system and to develop computational codes by a finite-element method in order to perform dynamics simulation on vibration of the manipulator system. The system used in this research consist of an aluminium beam as a flexible link, a clamp-part to hold the link, a DC motor and a lead screw set to move the link translationally. Computational codes on time history responses and FFT (Fast Fourier Transform) processing were developed to calculate dynamic behaviour of the link. The simulation result show the dynamic behaviour of the system on free vibration and forced vibration by excitation force due to the translational motion of the single-link manipulator system.

**Keywords:** Finite-element method, flexible manipulator, translational motion.

## 1. Introduction

In industrial applications and robotics system, a single-link flexible manipulator is expected to achieve maximum workability during operations such as high speed with safe operation, improved positioning accuracy, lighter weight, and lower energy consumption. Problems that often arise in flexible-link manipulator are vibrations caused by its flexibility which causes deformations that interfere with the performance of the system.

In the last decade, a number of researchers have investigated dynamic behaviour of flexible manipulator. Muhammad et al investigated dynamic of flexible manipulator with rotational motion using finite-element method including vibration control [1]–[8]. Muhammad et al used two node element in the investigation.

This study aims to formulate the equation of motion of the system and to develop computational codes with finite-element methods to find out time history responses and FFT (Fast Fourier Transform) processing of the link. The results of the plot time history response and FFT generated the natural frequency of the system.

Grounded in the previous work on computer simulation of a flexible single-link manipulator [8] and a flexible two-link manipulator [3] which failed to include the translational motion, this study put an emphasis on the translational motions in which these motions are often used in industrial applications and robotics. Another previous study also reported that the result of numeric simulation of a two-link



flexible manipulator was used to verify the control scheme effectively [9]. In relation to the current research, some similarities were also identified such as the use of a kinetic energy, potential energy, and Lagrange equation to derive the equation of motion[10]. Beyond the similarities, Bien's work focuses on translational and rotational joint of two-link manipulator while this piece of work focuses on translational motion with a clamp-part to hold the link.

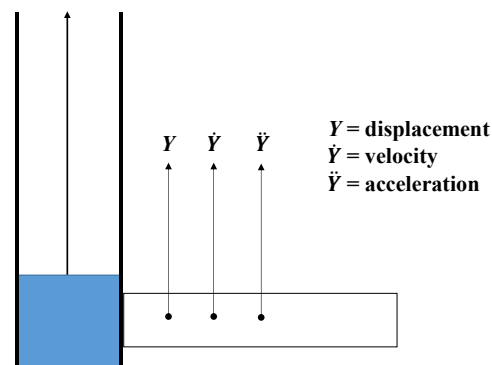
Prior to the application of active-force control on vibration of a single-link manipulator [2], it is necessary to have a clear picture of the characteristic of the link without vibration control. Obtaining the value of the free vibration is necessary to find out the natural frequency of the link. Furthermore, the natural frequency is pivotal to determine the type of effective control.

## 2. Formulation by finite-element method

Grounded in a finite-element method [11], each partition of the link is partially calculated to find out the matrix of mass ( $M$ ) and stiffness ( $K$ ) which generated a natural frequency of system. The link with a cantilever structure with a length of 33 cm is divided into 6 elements and move translationally by a DC motor to determine the vibration characteristics of the link. The degrees of freedom of the finite-element are divided into two types namely the lateral deformation  $v(t)$  and the rotational angle  $\Psi(t)$ . The physical properties of the system consists of the length ( $L_i$ ), the cross-sectional area ( $S_i$ ), and the area moment of inertia ( $I_i$ ). Each element of the mechanical properties (Young's modulus and mass density) are denoted as  $E_i$  and  $\rho_i$ .

### 2.1. Kinematics

Some text Figure 1 plots the directions of translational motion on the link into three types namely the displacement ( $Y$ ), velocity ( $\dot{Y}$ ), and acceleration ( $\ddot{Y}$ )

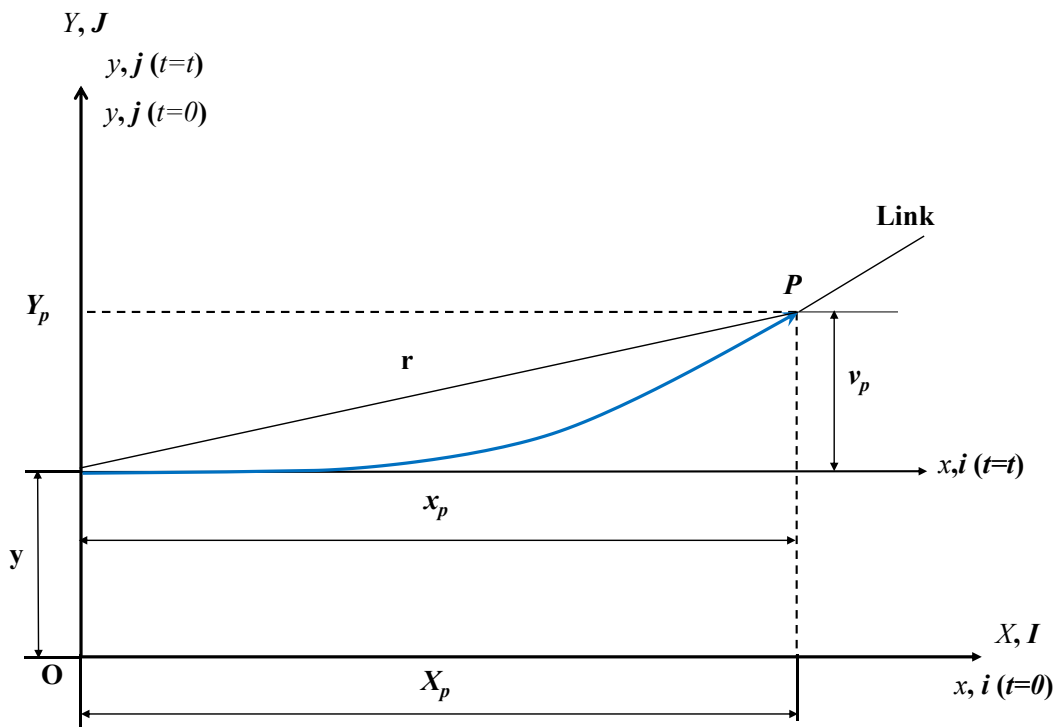


**Figure 1.** Directions of translational motion on the link

Figure 2 shows the position vector at point  $P$  on the link in the global coordinate frame and translating coordinate frame.  $O - XY$  is the global coordinate frame and  $O - yx$  is the translational coordinate frame. When the link move translationally, the displacement at  $X$  axis is  $x_p$  and the displacement at  $Y$  axis is  $Y_p$ . The deflection due to the translational motion is denoted as  $v_p$ .

The vector position  $r(x, t)$  at point  $P$  on the link at time  $t = t$ , in the global coordinate frame  $O - XY$  shown in the figure 2 is given by:

$$r(x, t) = X_p(x, t)\mathbf{I} + Y_p(x, t)\mathbf{J} \quad (1)$$



**Figure 2.** Position vector of an arbitrary point  $P$  of the link in the translational coordinate frames

The position vector of  $P$  in frame  $O - XY$  is

$$X_p(x, t) = x_p \tag{2}$$

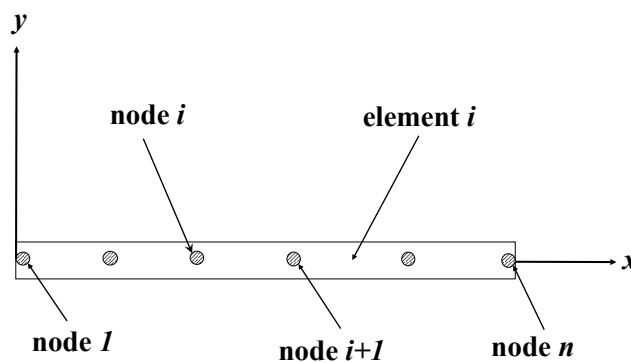
$$Y_p(x, t) = y + v_p \tag{3}$$

The velocity vector of  $P$  in frame  $O - xy$ , given by

$$\dot{\mathbf{r}}(x, t) = \dot{X}_p(x, t)\mathbf{I} + \dot{Y}_p(x, t)\mathbf{J} \tag{4}$$

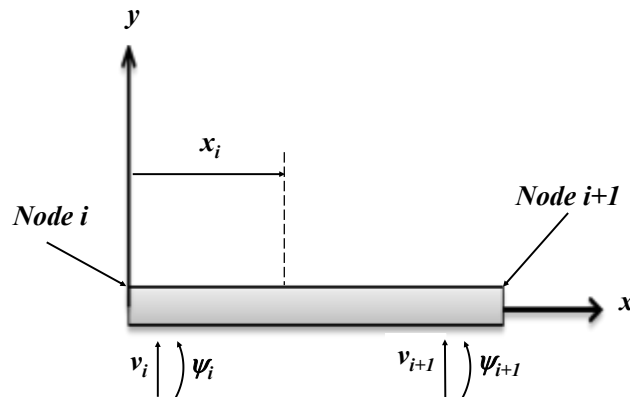
### 2.2. Finite-element method

Figure 3. illustrates the partitions of the link which are divided into six elements. Each node has the lateral deformation  $v(t)$  and the rotational angle  $\Psi(t)$ .



**Figure 3.** Translating coordinate frame of the link

Then, figure 4 shows the element coordinate frame of the *i*-th element.



**Figure 4.** Element coordinate frame of the *i*-th element

The nodal displacement vector is

$$\delta_i = \{v_i \quad \varphi_i \quad v_{i+1} \quad \varphi_{i+1}\}^T \tag{5}$$

### 2.3. Equation of motion

The discretization generated from the previous section, the velocity square of *P* can be obtained

$$\dot{\mathbf{r}}^T \cdot \dot{\mathbf{r}} = \dot{x}_p^2 + \dot{y}^2 + \dot{v}_p^2 + 2\dot{y}\dot{v}_p \tag{6}$$

The kinetic energy (*T<sub>i</sub>*) of system can be given by

$$\mathbf{T}_i = \int_{v_i} \rho_i \cdot \dot{\mathbf{r}}^T \cdot \dot{\mathbf{r}} \cdot dv_i \tag{7}$$

The kinetic energy (*T<sub>i</sub>*) of system can be expressed as

$$\mathbf{T}_i = \frac{1}{2} m_i \dot{x}_p^2 + \frac{1}{2} m_i \dot{y}^2 + \frac{1}{2} \dot{\delta}_i^T \mathbf{M}_i \dot{\delta}_i + \dot{y} \mathbf{f}_{ti}^T \dot{\delta}_i \tag{8}$$

Where

$$\dot{y} \mathbf{f}_{ti}^T \dot{\delta}_i = \frac{m_i}{12} [6 \quad l_i \quad 6 \quad -l_i]$$

The potential energy of the system can be written as

$$\mathbf{U}_i = \frac{1}{2} \delta_i^T K_i \delta_i \tag{9}$$

The dynamic model is formulated using the Lagrange equation can be defined by

$$M_i \ddot{\delta}_i - \dot{y}(t) \mathbf{f}_{ti}^T + C_i \dot{\delta}_i + K_i \delta_i = 0 \tag{10}$$

The equation of motion of the *i*-th element is given by:

$$M_i \ddot{\delta}_i + C_i \dot{\delta}_i + K_i \delta_i = \dot{y}(t) \mathbf{f}_{ti}^T \tag{11}$$

Where mass matrix ( $M_i$ ), damping matrix ( $C_i$ ), stiffness matrix ( $K_i$ ), and the excitation force of the translation motion ( $\ddot{y}(t)f_{ti}^T$ )

Matrix  $M_i$ ,  $C_i$ ,  $K_i$  and vector  $\ddot{y}(t)f_{ti}^T$  in Eq. (11) are represented as

$$M_i = \frac{\rho_i S_i L_i}{420} \begin{bmatrix} 156 & 22L_i & 54 & -13L_i \\ 22L_i & 4L_i^2 & 13L_i & -3L_i^2 \\ 54 & 13L_i & 156 & -22L_i \\ -13L_i & -3L_i^2 & -22L_i & 4L_i^2 \end{bmatrix} \quad (12)$$

$$K_i = \frac{E_i I_{zi}}{L_i^3} \begin{bmatrix} 12 & 6L_i & -12 & 6L_i \\ 6L_i & 4L_i^2 & -6L_i & 2L_i^2 \\ -12 & -6L_i & 12 & -6L_i \\ 6L_i & 2L_i^2 & -6L_i & 4L_i^2 \end{bmatrix} \quad (13)$$

$$C_i = \alpha \cdot K_i \quad (14)$$

$$f_{ti}^T = -\frac{\rho_i S_i L_i}{12} \{6, l_i, 6, -l_i\} \quad (15)$$

The length of the  $i$ -th element, the length from element 1 to  $i$ , and the Rayleigh damping factor are denoted by  $l_i$ ,  $l_{1-i}$ , and  $\alpha$  [12] respectively

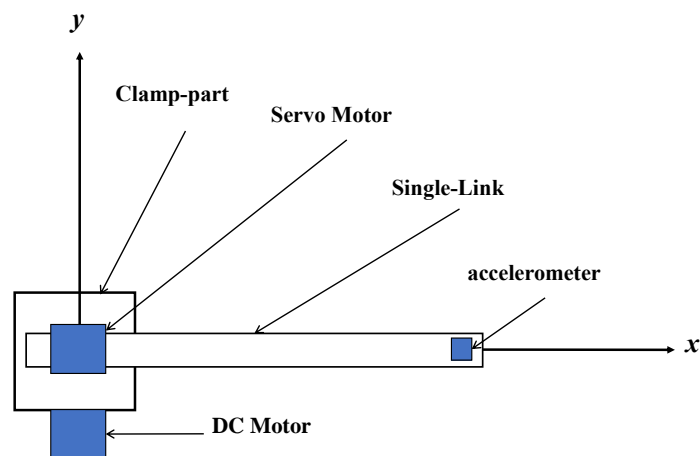
In the end, the equation of motion of the single-link with  $n$  element considering the boundary conditions is written by

$$M_n \ddot{\delta}_n + C_n \dot{\delta}_n + K_n \delta_n = \ddot{y}(t)f_{tn} \quad (16)$$

### 3. Dynamic behaviour of the system

#### 3.1. Computational model

Figure 5 illustrates the system of a flexible single-link manipulator which has a track of the link, clamp as the holder of the link, a servo motor to rotate the link, and a DC motor to make a translation motion.



**Figure 5.** Model system of flexible Single-link manipulator

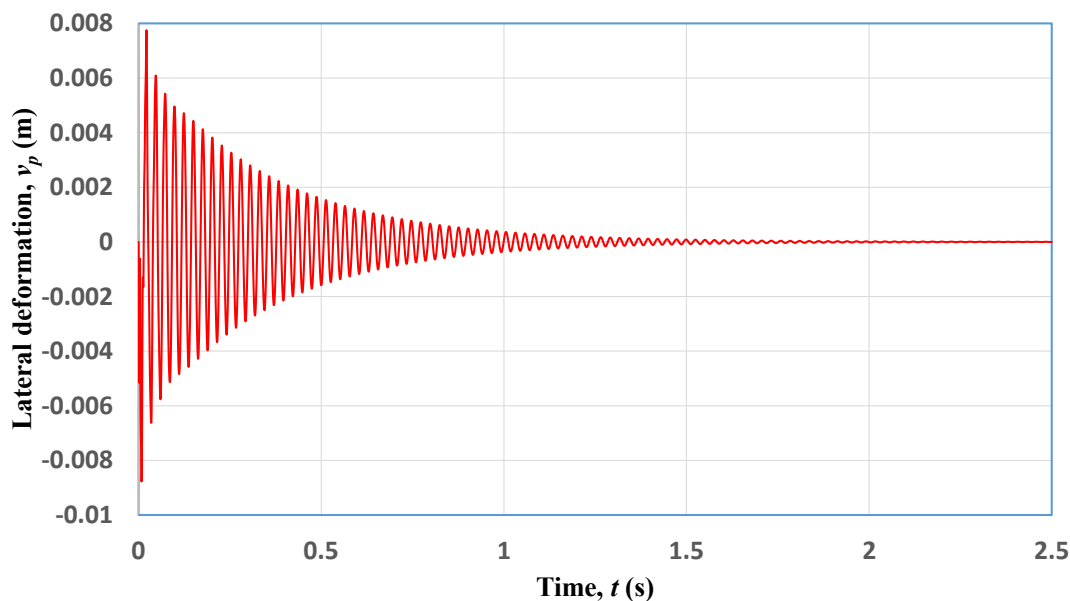
### 3.2. Physical parameters of dynamic models

**Table 1.** Physical parameters of the link

Property	Symbol	value
Total length (m)	$L$	$3.30 \times 10^{-1}$
Length of the link (m)	$l_i$	$3.00 \times 10^{-1}$
breadth of cross section (m)	$b_i$	$2.50 \times 10^{-2}$
height of cross section (m)	$h_i$	$1.00 \times 10^{-3}$
Cross section area of the link (m <sup>2</sup> )	$S_i$	$1.95 \times 10^{-5}$
Cross section area moment of inertia around $i$ -axis of the link (m <sup>4</sup> )	$I_i$	$2.75 \times 10^{-12}$
Young's Modulus of the link (GPa)	$E_i$	$7.00 \times 10^1$
Density of the link (kg/m <sup>3</sup> )	$\rho_i$	$2.70 \times 10^3$
Damping factor of the link	$\alpha$	$0.10 \times 10^{-3}$

### 3.3. Time history response on free vibration

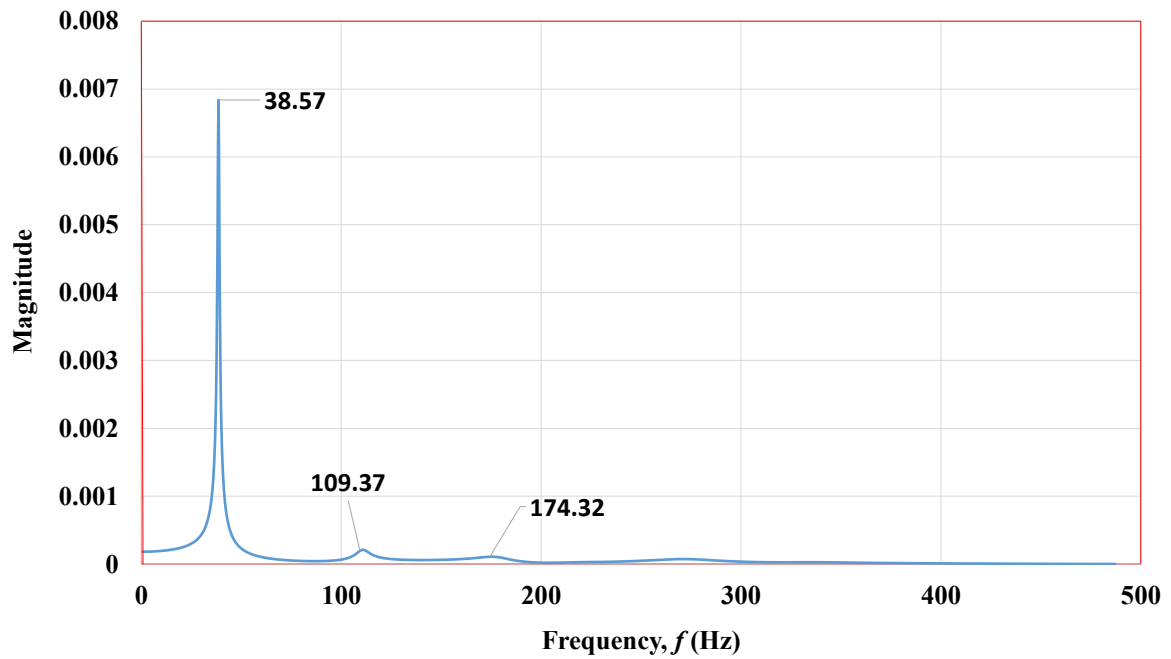
Time history response on free vibration simulation on free vibration was conducted using an impulse force as an external one. Figure 6 shows the simulation time history response of lateral deformation  $v_p$  on the free vibration. Furthermore, the computational codes on time history response of single-link were developed. Figure 6 shows the calculated lateral deformation at Node 6 of the system under the impulse force.



**Figure 6.** Time history response of the system

### 3.4. FFT (Fast Fourier Transform) Processing.

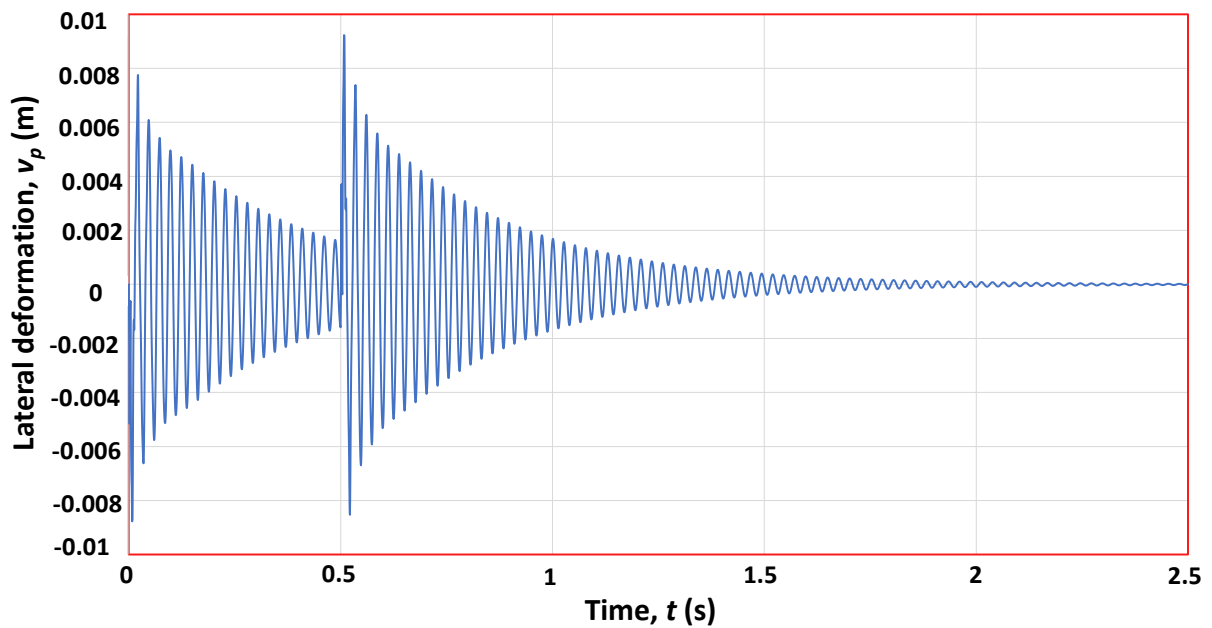
The calculated result from time history response to lateral deformation with free vibration on the system was transferred by FFT processing to find its frequencies. Figure 7 shows the calculated natural frequencies of the flexible link manipulator. The first, second, and third natural frequency are 38.57 [Hz], 109.37 [Hz], and 174.32 [Hz].



**Figure 7.** Calculated natural frequencies of the link

3.5. *Time history response of the system with excitation force.*

The calculated result from time history response to lateral deformation with excitation force on the system is shown in the figure 8. Showing at 0.5 (s) in the timeline, the lateral deformation becomes large due to the excitation force ( $f_t = 5$ ).



**Figure 8.** Time history response of the system with excitation force

#### 4. Conclusions

The equation of motion on the single-link flexible manipulator moved translationally has been discretized using a finite-element method. The computational codes were developed to simulate the dynamic system using the Scilab application. The simulation and calculated result of time history response, natural frequencies, and vibration mode showed validated formulation, computational codes, and model system. Fast Fourier Transform (FFT) has also generated the first, second, and third natural frequencies values as (f1) 38.57 [Hz], (f2)109.37 [Hz], and (f3)174.32 [Hz].

Further research recommends to combine the translational and the rotational motion on the single-link flexible manipulator by finding the equation of translational and rotational motion. The further step will continue doing an experiment of the model.

#### References

- [1] A. K. Muhammad, S. Okamoto, and J. H. Lee, "Comparison Between the One Piezoelectric Actuator and the Two Ones on Vibration Control of a Flexible Two-Link Manipulator Using Finite Element Method," *Int. J. Mech. Eng.*, vol. **5**, no. 1, pp. 25–42, 2016.
- [2] A. K. Muhammad, S. Okamoto, and J. H. Lee, "Finite Element Analysis for Active-force Control on Vibration of a Flexible Single-link Manipulator," *Int. J. Smart Mater. Mechatronics*, vol. **2**, no. April, p. 18, 2016.
- [3] A. K. Muhammad, S. Okamoto, and J. H. Lee, "Computer Simulations and Experiments on Vibration Control of a Flexible Link Manipulator Using a Piezoelectric Actuator Computer Simulations and Experiments on Vibration Control of a Flexible Link Manipulator Using a Piezoelectric Actuator," *Eng. Lett.*, vol. **3**, no. 23, 2015.
- [4] A. K. Muhammad, S. Okamoto, and J. H. Lee, "Active-Force Control on Vibration of a Flexible Single-Link Manipulator Using a Piezoelectric Actuator," *Trans. Eng. Technol.*, pp. 1–15, 2015.
- [5] A. K. Muhammad, S. Okamoto, and J. H. Lee, "Active-Force Control on Vibration of a Flexible Single-Link Manipulator Using a Piezoelectric Actuator," in *Transactions on Engineering Technologies*, springer, 2015, pp. 1–15.
- [6] A. K. Muhammad, S. Okamoto, and J. H. Lee, "Comparison of proportional-derivative and active-force controls on vibration of a flexible single-link manipulator using finite-element method," *Artif. Life Robot.*, vol. **19**, no. 4, pp. 375–381, 2014.
- [7] A. K. Muhammad, S. Okamoto, and J. H. Lee, "Comparisons of proportional and active-force controls on vibration of a flexible link manipulator using a piezoelectric actuator through calculations and experiments," *Eng. Lett.*, vol. **22**, no. 3, p. 8, 2014.
- [8] A. K. Muhammad, S. Okamoto, and J. H. Lee, "Computer simulations on vibration control of a flexible single-link manipulator using finite- element method," in *The Nineteenth International Symposium on Artificial Life and Robotics 2014*, 2014.
- [9] X. Yang and Z. Zhong, "Dynamics and Terminal Sliding Mode Control of Two-Link Flexible Manipulators with Noncollocated Feedback," *IFAC Proc. Vol.*, vol. **46**, no. 20, pp. 218–223, 2013.
- [10] D. X. Bien, C. A. My, and P. B. Khoi, "Dynamic Modeling and Control of a Flexible Link Manipulators with Translational and Rotational Joints," *VNU J. Sci. Math. – Phys.*, vol. **34**, no. 1, pp. 52–66, 2018.
- [11] S. S. Rao, *The finite element method in engineering*, Fifth Edit. Elsevier, 2011.
- [12] M. Lalanne, P. Berthier, and J. Der Hagopian, *Mechanical Vibrations for Engineers*. <cel-00315803> HAL, 1984.